Exact controllability to the trajectories of the one-phase Stefan problem

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Abstract

In this talk we are going to study the boundary exact controllability to the trajectories of the one-phase Stefan problem in one spatial dimension. This is a free-boundary problem that models solidification and melting processes. It is assumed that the physical domain is filled by a medium whose state is liquid on the left and solid, with constant temperature equal to zero, on the right. In between we find a free-boundary (the interface that separates the liquid from the solid). In the liquid domain, a parabolic equation completed with initial and boundary conditions must be satisfied by the temperature. On the interface, an additional equality, called the Stefan condition, is imposed. We prove the local exact controllability to the (smooth) trajectories. To this purpose, we first reformulate the problem as the local null controllability of a coupled PDE-ODE system with distributed controls. Then, a new Carleman inequality for the adjoint of the linearized PDE-ODE system, coupled on the boundary through nonlocal in space and memory terms, is presented. This leads to the null controllability of an appropriate linear system. Finally, the result is obtained via local inversion, by using Liusternik-Graves' Theorem. As a byproduct of our approach, we find that some parabolic equations which contains memory terms located on the boundary are null-controllable.

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