One-dimensional wave equations with set-valued boundary damping*

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Abstract

The asymptotic behavior of linear wave equations with constant coefficients in one space dimension is highly dependent on their boundary conditions: homogeneous Dirichlet or Neumann boundary conditions yield conservation of energy, whereas a transparent boundary condition, for instance, provides convergence to 0 in finite time. In this talk, we shall consider more general boundary conditions described by set-valued maps. After an introduction summarizing important previous works and explaining the interest of more general boundary conditions in some applications, we will explain why set-valued boundary conditions represent, from the theoretical point of view, a very natural framework for boundary conditions of wave equations. In particular, we provide necessary and sufficient conditions for the existence and uniqueness of solutions. Our results contain some highly nontrivial facts, such as the fact that uniqueness may hold even if the set-valued map describing the boundary condition is not a function, and that uniqueness may fail to hold even if the set-valued boundary condition is a function.

After discussing such well-posedness issues, we turn our attention to asymptotic behavior. We will show how our techniques allow us to retrieve some known results on the asymptotic behavior of wave equations with several different boundary conditions and provide answers to previously open questions, describing the decay rate of solutions for several families of boundary conditions.

∗ This talk is based on joint works with Yacine Chitour and Guilherme Mazanti.