Ulysseus Spring School in PDEs - USSPDE
Universidad de Sevilla and Université Côte d'Azur
Seville, June 12-16, 2023

Mathematical modelling and numerical analysis in chemotaxis

Francisco Guillén González and María Ángeles Rodriguez Bellido*

The interaction between living organisms (cells for instance) and chemical signals are modelled by the known as chemotaxis models, which are Parabolic PDEs system where the spatial transport due to chemotaxis introduces a nonlinear second order term. This mini-course will be oriented in two main directions:

- Mathematical modelling in chemotaxis: Several types of models will be presented, where different effects are taking into account, as diffusion (in each variable), chemotaxis (spatial movement of cells either towards the chemical signal, in the attractant case, or in the opposite sense, in the repulsion case), growth logistic term for cells, production or consumption of chemical by cells. Normally, all these effects are put together in a bounded spatial domain, imposing isolated boundary conditions. Then, several properties of these models will be presented, as conservation, nonnegativity, specific energy functionals which can be decreasing along the trajectories or at least bounded. These properties are essential to derive some analytical results as: existence of global weak solutions, large time behavior, existence (and unicity) of global classical solutions (in some cases) and blow-up solutions (due to strong aggregation effects) (in other cases).
- Numerical analysis in chemotaxis: The idea of this part is to present the main numerical schemes existing in the literature approaching chemotaxis problems. In the translation from continuous (infinite dimension) PDEs problems to fully discrete problem (which can be computed numerically) some properties of the PDEs problem could be lost. Therefore, the main task is to design fully discrete numerical schemes conserving as many properties as possible. Several schemes have been designed using different strategies, as Finite Difference (FD), Finite Element (FE), Finite Volume (FV), and Discontinuos Galerkin (DG). In this course some of these schemes will be presented, discussing their properties, letting to finish with a comparison between all schemes.

*Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla