Ulysseus Spring School in PDEs

Institute of Mathematics of the University of Seville (IMUS)

June 12-16, 2023

Book of Abstracts

Ulysseus Spring School in PDEs - USSPDE

Organizing committee

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Thierry Goudon (UCA - France)
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Welcome

On behalf of the Organizing Committee, it is a pleasure to welcome all the participants in the Ulysseus Spring School in PDEs, to be held on June from 12 to 16, 2023 at the Institute of Mathematics of the University of Seville.

This Ulysseus Spring School in PDEs - USSPDE is a mathematical meeting in the spirit of the Ulysseus Consortium, aimed at bringing together researchers and students from several Ulysseus partners, in particular the Université Côte d'Azur (LJAD) and the Universidad de Sevilla (EDAN, IMUS). The goal is to present and share recent research results on PDEs in a pleasant atmosphere which, hopefully, will help to strengthening cooperation links.

The members of the organizing committee wish to express their gratitude to the institutions that have supported and made possible the realization of this event.

It is our hope that this meeting be followed by other Ulysseus workshops and schools and consequently contribute to create a rich and fruitful frame of work and collaboration.

The Organizing Committee

ULYSSEUS SPRING SCHOOL IN PDES

Ulysseus
EUROPEAN UNIVERSITY

June 12-16, 2023
Institute of Mathematics of the
University of Seville (IMUS)

Scientific committee

Blanca Climent Ezquerra
Enrique Domingo Fernández Nieto
Thierry Goudon
Roland Masson

Organization

Laboratoire J. A. Dieudonné
&
Departamento de Ecuaciones
Diferenciales y Análisis Numérico

Activities

Mini-courses, Plenary Talks,
Poster Session, Roundtable

Invited speakers

Jean-Baptiste Caillau
Juan Casado Díaz
Yves D'Angelo
Enrique Delgado Ávila
Alessandro Felisi
Enrique Fernández-Cara
Francisco Guillén González
Stéphane Junca
José Antonio Langa Rosado
Faustino Maestre Caballero
Florence Marcotte
Laurent Monasse
Cristian Morales Rodrigo
María Ángeles Rodríguez-Bellido
Simona Rota-Nodari

For more information you can scan the
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http://departamento.us.es/edan/USSPDE23/

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Ulysseus Spring School in PDEs - June 12th to 16th, 2023

Location: Main Lecture Hall (ground floor), Instituto Matemáticas de la Universidad Sevilla (IMUS)

	Monday 12	Tuesday 13
8:00-8:30	Registration	
8:30-9:00	Opening ceremony	
9:00-10:00	**Yves D'Angelo** *Chairman: Iván Moyano*	**José A. Langa** *Chairman: Pedro Marín-Rubio*
10:00-11:00	**Laurent Monasse** *Chairman: Iván Moyano*	**Alessandro Felisi** *Chairman: Pedro Marín-Rubio*
11:00-12:00	Coffee-Break	Coffee-Break/Poster Session
12:00-13:00	**Jean-Baptiste Caillau** *Chairman: Iván Moyano*	**Cristian Morales-Rodrigo** *Chairman: Pedro Marín-Rubio*
13:00-15:00	Lunch	Lunch
15:00-16:30	**Juan Casado-Díaz** **and** **Faustino Maestre** *Chairman: Manuel Luna-Laynez*	**Juan Casado-Díaz** **and** **Faustino Maestre** *Chairman: Blanca Climent Ezquerra*

	Wednesday 14	Thursday 15
9:00-10:30	**Simona Rota-Nodari** *Chairman Mayte Pérez-Llanos*	**Simona Rota-Nodari** *Chairman: Silvia Sastre-Gómez*
10:30-11:30	Coffee-Break/Poster Session	Coffee-Break
11:30-13:00	**Francisco Guillén-González** **and** **María Ángeles Rodríguez-Bellido** *Chairman Mayte Pérez-Llanos*	**Francisco Guillén-González** **and** **María Ángeles Rodríguez-Bellido** *Chairman: Silvia Sastre-Gómez*
13:00-13:15	Official Photo	Lunch
13:15-15:00	Lunch	
15:00-16:30	Free Afternoon	**Roundtable:** *Cooperation, opportunities, and future collaborations*
16:30-20:45		
20:45-22:30	Cocktail reception *At Mercado Lonja del Barranco*	

	Friday 16
9:00-10:00	**Stephane Junca** *Chairman: Antonio Suárez Fernández*
10:00-11:00	**Enrique Delgado Ávila** *Chairman: Antonio Suárez Fernández*
11:00-12:00	Coffee-Break
12:00-13:00	**Enrique Fernández-Cara** *Chairman: Antonio Suárez Fernández*
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13:15-15:00	Lunch

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Plenary Talks

Ulysseus Spring School in PDEs - USSPDE
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Seville, June 12-16, 2023

Optimisation of Sturm-Liouville determinants

Jean-Baptiste Caillau[1]

Consider the laplacian + potential operator on the circle, and try to optimise its "functional" determinant wrt. essentially bounded potentials. I will show that this question is related to the spectral zeta function, to a particular dynamics on $SL(2, R)$, and to bilinear optimal control. Joint work with Y. Chitour, P. Freitas et Y. Privat.

[1]Laboratoire Jean Alexandre Dieudonné, Université Côte d'Azur

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DREAMS: an interdisciplinary project. Dynamics of Random ExpAnding MultiScale networks

Yves D'Angelo[2]

I shall first present the biology of fungal networks: filamentous fungi, apexes, hyphae, thallus, mycelium, give many examples and insist on the multi-scale aspects : from the nanoscale up to the very large scale. An essential diversion deals with combustion, flames, reactive flows, active fronts, which is also a multi-scale phenomenon. I will then draw a parallel between these two topics: random expanding (discrete) networks may yield active fronts. Finally, I will mention some possible other applications.

[2]Laboratoire Jean Alexandre Dieudonné, Université Côte d'Azur

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An overview on Reduced Order for Large Eddy Simulation turbulence models

Enrique Delgado-Ávila[3]

In this talk, we present an overview of Reduced Order Modelling (ROM) for fluid flows based upon Large Eddy Simulation (LES) turbulence models, in particular, the Smagorinsky model. We present different approaches for the construction of the reduced order models. We present some Reduced Basis Models for which a posteriori error estimators are developed in order to select properly the basis functions. Different strategies for recovering the reduced pressure are presented. Moreover, non-linearities of the Full Order Model (FOM) must be treated in an efficient way. For this purpose, we present strategies to linearize the non-linear terms that comes from the FOM. Finally, some numerical results are presented for different LES turbulence models.

[3]Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Full discretization and regularization for the Calderón problem

Alessandro Felisi[4]

In this talk, I will introduce the inverse conductivity problem for discontinuous conductivities, also known as the Calderón problem, which is an example of a parameter identification problem for elliptic PDEs. It is possible to recast it as a minimization problem whose solution is a good approximation of a solution to the original inverse problem. The objective functional contains a regularization term which is given by a total variation penalization and is characterized by a regularization parameter. The discretization involves at the same time the boundary measurements, the unknown conductivity and the solution to the direct problem. I will show how to precisely choose the regularization, electrodes size and mesh size parameters with respect to the noise level in such a way that the solution to the discrete regularized problem is meaningful. This is joint work with Luca Rondi (University of Pavia).

[4]Dipartimento di Matematica, Università degli Studi di Genova

On theoretical and numerical control and inverse problems

Enrique Fernández-Cara[5]

This talk is devoted to recall several control and inverse problems that we have been considering since more than thirty years. For reasons of time, we will give details only on contributions concerning

- The null controllability of Navier-Stokes and some similar systems.
- The boundary controllability properties of non-scalar systems.
- Control and inverse problems for free-boundary systems.
- Bi-objective and hierarchical control problems.
- Controlling equations with nonlocal terms in time and space.

We will also indicate several open problems that can motivate our work in the next future. The contributions have been obtained in collaboration with several people, among them A. Doubova, M. González-Burgos, I. Marín-Gayte and D.A. Souza.

[5]Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

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Fractional BV spaces for hyperbolic conservation laws

Stéphane Junca[6]

The theory of weak entropy solutions with shock waves has usually performed in the space BV of functions of bounded total variation or in L^∞. What happens between BV and L^∞? The spaces, BV^s, $0 < s < 1$ fill the gap between $BV = BV^1$ and L^∞. They have some BV-properpties, they include shock wave functions, traces and compactness embedding in L^1_loc. Some simple examples are first presented in BV^s, a Lions-Perthame- Tadmor conjecture, existence or blow-up of solutions, and some expected results.

[6]Laboratoire Jean Alexandre Dieudonné, Université Côte d'Azur

Structural stability of infinite-dimensional dynamical systems. Some applications to real phenomena

José Antonio Langa Rosado[7]

In the first part of this talk we will present a review on main results on structural stability of dynamical systems associated to PDEs. The stability of the attractor characterization under autonomous and non-autonomous perturbation is one of the main topics pointing the importance of global attractors for the analysis of dissipative differential equations. In particular, we will introduce the concepts of informational structures and fields as proper tools to analyze dynamics on cooperative complex networks. Some applications to real phenomena in Theoretical Ecology and Neuroscience will be addressed.

[7]Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Fisher/KPP models with memory for fungal growth and their numerical simulation

Laurent Monasse[8]

Filamentous fungi form interconnected networks which enable them to explore, exploit and efficiently share resources on large scales. In order to understand the mechanics behind network formation and characterize its behavior, we propose a model based on three basic mechanisms: growth under a Langevin dynamics, branching at the tips and on the network, and anastomosis by fusion of crossing filaments. In the limit of large scales with regards to the filament size, we can model the homogenized density as a hyperbolic isothermal Euler equation with nonlinear reaction terms. In the limit of large temperature, the system boils down to a Fisher/KPP model with memory. We are able to analyze the asymptotic velocity of the front in both full and simplified models. However, from a numerical point of view, the handling of parabolic Fisher/KPP equations is very different (and easier) from the full hyperbolic model. For the full model, we use an Asymptotics-preserving approach to accurately recover the correct front propagation velocity.

[8]Laboratoire Jean Alexandre Dieudonné, Université Côte d'Azur

Nonlocal and Interface problems

Cristian Morales-Rodrigo[9]

In this talk we will consider various linear and nonlinear problems mostly of elliptic type with nonlocal terms or interface problems for nonsymmetric operators. We will focus mainly on nonlinearities of logistic type.

[9] Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Minicourses

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Optimal design through the homogenization theory

Juan Casado-Díaz and Faustino Maestre Caballero[10]

We consider a control problem for an elliptic PDE, where the control variables are given by the coefficients of the equation, and the open set where the equation is posed. This type of problems arises in the optimal design of materials and shapes. It is well known the lack of solution, which makes necessary to introduce a relaxed formulation. Here, we show how this can be done using the homogenization theory. Moreover, from the numerical point of view, this relaxed formulation has better properties than the original one. In this sense, we will present a gradient descent algorithm and we apply it to solve some examples.

[10]Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

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Mathematical modelling and numerical analysis in chemotaxis

Francisco Guillén González and María Ángeles Rodríguez-Bellido[11]

The interaction between living organisms (cells for instance) and chemical signals are modelled by the known as chemotaxis models, which are Parabolic PDEs system where the spatial transport due to chemotaxis introduces a nonlinear second order term. This mini-course will be oriented in two main directions:

- Mathematical modelling in chemotaxis: Several types of models will be presented, where different effects are taking into account, as diffusion (in each variable), chemotaxis (spatial movement of cells either towards the chemical signal, in the attractant case, or in the opposite sense, in the repulsion case), growth logistic term for cells, production or consumption of chemical by cells. Normally, all these effects are put together in a bounded spatial domain, imposing isolated boundary conditions. Then, several properties of these models will be presented, as conservation, nonnegativity, specific energy functionals which can be decreasing along the trajectories or at least bounded. These properties are essential to derive some analytical results as: existence of global weak solutions, large time behavior, existence (and unicity) of global classical solutions (in some cases) and blow-up solutions (due to strong aggregation effects) (in other cases).
- Numerical analysis in chemotaxis: The idea of this part is to present the main numerical schemes existing in the literature approaching chemotaxis problems. In the translation from continuous (infinite dimension) PDEs problems to fully discrete problem (which can be computed numerically) some properties of the PDEs problem could be lost. Therefore, the main task is to design fully discrete numerical schemes conserving as many properties as possible. Several schemes have been designed using different strategies, as Finite Difference (FD), Finite Element (FE), Finite Volume (FV), and Discontinuos Galerkin (DG). In this course some of these schemes will be presented, discussing their properties, letting to finish with a comparison between all schemes.

[11] Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Introduction to nonlinear dispersive equations

Simona Rota–Nodari[12]

The aim of this mini–course is to present the analytical tools needed to discuss local and global existence results for (non)linear dispersive equations. Special attention will be given to nonlinear Schrödinger equations (NLS) and their applications.

[12]Laboratoire Jean Alexandre Dieudonné, Université Côte d'Azur

Posters

Ulysseus Spring School in PDEs - USSPDE
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Advances in obtaining optimal intrinsic PGD modes

Alejandro Bandera, Soledad Fernández-García and Macarena Gómez-Mármol[13]

In this work, we introduce an iterative optimization algorithm to obtain the intrinsic Proper Generalized Decomposition modes [1] of elliptic partial differential equations. The main idea behind this procedure is to adapt the general Gradient Descent algorithm to the algebraic version of the intrinsic Proper Generalized Decomposition framework, and then to couple a one-dimensional case of the algorithm with a deflation algorithm in order to keep enriching the solution by computing further intrinsic Proper Generalized Decomposition modes. For this novel iterative optimization procedure, we present some numerical tests based on physical parametric problems, in which we obtain very promising results in comparison with the ones already presented in the literature [2]. This support the use of this procedure in order to obtain the intrinsic PGD modes of parametric symmetric problems.

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[2] Azaïez, M., Rebollo, T. C. and Mármol, M. G., On the computation of proper generalized decomposition modes of parametric elliptic problems, *SeMA Journal*, 77 ,59–72 (2020).

[13]Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Kirchhoff-Boussinesq type problems with positive and zero mass

Romulo Díaz Carlos, Giovany M. Figuereido and Ricardo Ruviario[14]

In this presentation we will treat the question existence of solution for the following class of elliptic Kirchhoff-Boussinesq type problems given by

$$\Delta^2 u - \Delta_p u + u = h(u) \quad \text{in } \mathbb{R}^N \quad \text{and} \quad \Delta^2 u - \Delta_p u = f(u) \ \text{ in } \mathbb{R}^N,$$

where $2 < p \leq \frac{2N}{N-2}$ for $N \geq 3$ and $2_{**} = \infty$ for $N = 3$, $N = 4$, $2_{**} = \frac{2N}{N-4}$ for $N \geq 5$ and h and f are continuous functions that satisfy hypotheses considered by Berestycki and Lions in [2]. More precisely, the problem with the nonlinearity h is related to Positive mass case and the problem with the nonlinearity f is related to Zero mass case. The main argument is to find a Palais–Smale sequence satisfying a property related to Pohozaev identity, as in [4], which was used for the first time by [6], for more details you can see [3].

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[3] Carlos, R. D., Figueiredo, G. M. and Ruviaro, R. Kirchhoff–Boussinesq–type problems with positive and zero mass *Applicable Analysis*, (2023).

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[5] Hu, D. and Zhang, Q. Existence ground state solutions for a quasilinear Schrödinger equation with Hardy potential and Berestycki–Lions type conditions *Appl. Math. Lett.*, 123, Paper No. 107615 (2022).

[6] Jeanjean, L. On the existence of bounded Palais–Smale sequences and application to a Landesman–Lazer–type problem set on $\mathbb{R}^N$ *Proc. Roy. Soc. Edinburgh Sect. A*, 129 (4), 787–809 (1999).

[14] Departamento de Matemática, Universidade de Brasília, Brazil.

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Quasilinear elliptic systems involving the 1-Laplacian operator with subcritical and critical nonlinearities

Yino Carranza and Marcos T. O. Pimenta[15]

In this work, we study the systems of equations involving the 1–Laplacian operator. In the first part, we deal with the following system 1-Laplacian equations with subcritical growth:

$$\begin{cases} \text{-div}\left(\frac{Du}{|Du|}\right) = F_u(x,u,v) & \text{in} \quad \Omega, \\ \text{-div}\left(\frac{Dv}{|Dv|}\right) = F_v(x,u,v) & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $N \geq 2$, $\Omega \subset \mathbb{R}^N$ is an open bounded set, and F a function satisfying some hypotheses. In the second part of this work, we study the following system of elliptic equations with critical growth:

$$\begin{cases} \text{-div}\left(\frac{Du}{|Du|}\right) = Q_u(u,v) + \lambda\frac{2\alpha}{\alpha+\beta}u|u|^{\alpha-2}|v|^{\beta} & \text{in} \quad \Omega, \\ \text{-div}\left(\frac{Dv}{|Dv|}\right) = Q_v(u,v) + \lambda\frac{2\beta}{\alpha+\beta}|u|^{\alpha}v|v|^{\beta-2} & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \\ u,v \geq 0; \quad u,v \neq 0 & \text{in} \quad \Omega, \end{cases} \tag{2}$$

where $\alpha + \beta = 1^*$, $\lambda > 0$, $N \geq 2$, $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$ and Q_u, Q_v are the partial derivates of C^1-function Q. Our main result are the following:

Theorem 1. *Suppose that F satisfies appropriate growth conditions then, system (1) has a nontrivial solution.*

Theorem 2. *Let Q be a function $C^1(\mathbb{R}_+ \times \mathbb{R}_+; \mathbb{R})$ satisfying some hypotheses. Then, system (2) has a nontrivial solution.*

In both cases the solutions are obtained as limit of solutions to p–Laplacian type problems.

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[3] Salas, A. M. and Segura de León, S. Elliptic equations involving the 1-Laplacian and a subcritical source term *Nonlinear Analysis, Elsevier* , 168, 50–66 (2017).

[15] Instituto de Biociências, Letras e Ciências Exatas, Universidade Estadual Paulista, Brazil.

Unbounded attractors for non-dissipative semigroups

Juan Garcia Fuentes[16]

Dynamical systems governed by dissipative semigroups contain structures that are invariants and attracts every trajectory of the phase space, well know as global attractors, and by its own definition of dissipation, these global attractors are bounded sets. Nevertheless, in case of working with slowly non–dissipative semigroups, that is, its solutions can diverge to infinity as time tends to infinity, one can find also an invariant attracting structure, but in this case unbounded. We follow the study of Chepyzhov and Goritskii [1], to provide abstract results on the unbounded attractor existence. Furthermore, we study the properties of these attractors and the ω-limit sets for slowly non–dissipative semigroups. Finally, we illustrate the abstract results by the study of the autonomous problem governed by the equation $u_t = Au + f(u)$. This is a joint work with Jakub Bańaskiewicz, Alexandre Nolasco de Carvalho and Piotr Kalita, full developed in [2].

References

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[2] Banaskiewicz, J., Carvalho, A. N., Garcia–Fuentes, J. and Kalita, P., Autonomous and Non-autonomous Unbounded Attractors in Evolutionary Problems, *Journal of Dynamics and Differential Equations*, (2022).

[16]Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Study of maximal biodiversity in Lotka-Volterra competitive systems with higher-order interaction

Manuel Miranda and Antonio Suárez[17]

The competitive Lotka–Volterra system of ordinary differential equations has been intensively studied in recent years. In the present poster we will focus on this system with a higher-order term added

$$\begin{cases} u' = u(b_1 - u - a_{12}v) \\ v' = v(b_2 - v - a_{21}u - huv) \end{cases} \tag{3}$$

where $b_i \in \mathbb{R}$, $a_{ij} > 0$ and $h \geq 0$. Observe that in the classical Lotka–Volterra model, $h = 0$, the competition of the species v is described by a linear term $-a_{21}u$. However, recent works (see [3], [4] and references therein) describe observed species with non–linear effects of competition. It is well-known that when $h = 0$ then if

$$(b_1, b_2) \in \mathcal{R} := \{(b_1, b_2) \in \mathbb{R}^2_+ : a_{12}^{-1} b_1 > b_2 > a_{21} b_1\}$$

then there exists a unique coexistence state $(u^*, v^*) > (0, 0)$ globally stable, hence $\mathcal{R}$ is the maximal biodiversity region. We will ask if the inclusion of this new nonlinear term changes the structure of the maximum biodiversity domain. We will see that when h increases, then the region of maximum biodiversity increases. Specifically, in the case $h > 0$, we prove the existence of a region $\mathcal{R}_h$ with $\mathcal{R} \subset \mathcal{R}_h$, such that, if $(b_1, b_2) \in \mathcal{R}_h$ then there exists at least a locally asymptotically stable coexistence state. Finally, we also show that if $(b_1, b_2) \in \mathcal{R}_h \setminus \mathcal{R}$ two saturated equilibria exist: one is stable and the other is a saddle point. In this subregion, we demonstrate the existence of a separatrix curve, as is the case for $h = 0$ (see [2]), which separates the basin of attraction of the stable saturated steady state, which is always located below the curve, and the basin of attraction of the semi-trivial state located above the curve, which always exists in this case. This result is an extension of [1] for $h > 0$.

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[2] Langa, J. A., Robinson, J. C. and Suárez, A. Forwards and pullback behaviour of a non-autonomous Lotka-Volterra system. *Nonlinearity*, 16 (4), 1277–1293 (2003).

[3] Letten, A. D. and Stouffer, D. B. The mechanistic basis for higher-order interactions and non-additivity in competitive communities. *Ecology Letters*, 22 (3), 423–436 (2019).

[4] Mayfield, M. M. and Stouffer, D. B. Higher-order interactions capture unexplained complexity in diverse communities. *Nat. Ecol. Evol.*, 1, paper No 0062 (2017).

[17] Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla

Continuum of solutions from a continuation theorem on open sets

Vinicius Kobayashi Ramos, Carlos Alberto Santos and Willian Cintra[18]

In this poster we will present a result that provides the existence of a continuum of positive solutions (λ, u) of $u = K(\lambda, u)$, emanating from a point (λ_0, u_0) with non zero Leray Schauder Index, where K is a compact operator defined on $\overline{\mathcal{U}}, \mathcal{U}$ is an open subset of $\mathbb{R} \times E$ (E Banach space) and u_0 is an isolated solution of $u = K(\lambda_0, u)$. The result is an improvement of Theorem 2.2 of [1] which requires the set of solutions for $\lambda = \lambda_0$ to be unitary and $\mathcal{U} = \mathbb{R} \times E$. By applying the result for $\lambda_0 = 0$ and an appropriated $\mathcal{U}$, we prove that the problem

$$\begin{cases} -\Delta u - \lambda u \Delta(u^2) = \mu u - u^p & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with $\mu > \lambda_1$ and $p > 1$, admits a positive solution for each $\lambda > -1/\left(2\mu^{\frac{2}{p-1}}\right)$. Also we prove some existence and qualitative information about positive solutions of a Kirchhoff-Carrier-type problem.

References

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[18]Departamento de Matemática, Universidade de Brasília

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