Ulysseus Spring School in PDEs - USSPDE
Universidad de Sevilla and Université Côte d'Azur
Seville, June 12-16, 2023

Quasilinear elliptic systems involving the 1-Laplacian operator with subcritical and critical nonlinearities

Yino Carranza and Marcos T. O. Pimenta[15]

In this work, we study the systems of equations involving the 1–Laplacian operator. In the first part, we deal with the following system 1-Laplacian equations with subcritical growth:

$$\begin{cases} \text{-div}\left(\frac{Du}{|Du|}\right) = F_u(x,u,v) & \text{in} \quad \Omega, \\ \text{-div}\left(\frac{Dv}{|Dv|}\right) = F_v(x,u,v) & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \end{cases} \tag{1}$$

where $N \geq 2$, $\Omega \subset \mathbb{R}^N$ is an open bounded set, and F a function satisfying some hypotheses. In the second part of this work, we study the following system of elliptic equations with critical growth:

$$\begin{cases} \text{-div}\left(\frac{Du}{|Du|}\right) = Q_u(u,v) + \lambda\frac{2\alpha}{\alpha+\beta}u|u|^{\alpha-2}|v|^{\beta} & \text{in} \quad \Omega, \\ \text{-div}\left(\frac{Dv}{|Dv|}\right) = Q_v(u,v) + \lambda\frac{2\beta}{\alpha+\beta}|u|^{\alpha}v|v|^{\beta-2} & \text{in} \quad \Omega, \\ u = v = 0 & \text{on} \quad \partial\Omega, \\ u, v \geq 0; \quad u, v \neq 0 & \text{in} \quad \Omega, \end{cases} \tag{2}$$

where $\alpha + \beta = 1^*$, $\lambda > 0$, $N \geq 2$, $\Omega \subset \mathbb{R}^N$ is a bounded domain with smooth boundary $\partial\Omega$ and Q_u, Q_v are the partial derivates of C^1-function Q. Our main result are the following:

Theorem 1. *Suppose that F satisfies appropriate growth conditions then, system* (1) *has a nontrivial solution.*

Theorem 2. *Let Q be a function $C^1(\mathbb{R}_+ \times \mathbb{R}_+; \mathbb{R})$ satisfying some hypotheses. Then, system* (2) *has a nontrivial solution.*

In both cases the solutions are obtained as limit of solutions to p–Laplacian type problems.

References

[1] Demengel, F. On some nonlinear partial differential equations involving the "1"-Laplacian and critical Sobolev exponent *ESAIM: Control, Optimisation and Calculus of Variations, EDP Sciences* , 4, 667–686 (1999).

[2] de Morais Filho, D. C. and Souto, M. A. S. Systems of p-laplacean equations involving homogeneous nonlinearities with critical sobolev exponent degrees *Communications in partial differential equations*, 24 (7–8),1537–1553 (1999).

[3] Salas, A. M. and Segura de León, S. Elliptic equations involving the 1-Laplacian and a subcritical source term *Nonlinear Analysis, Elsevier* , 168, 50–66 (2017).

[15]Instituto de Biociências, Letras e Ciências Exatas, Universidade Estadual Paulista, Brazil.