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Study of maximal biodiversity in Lotka-Volterra competitive systems with higher-order interaction

Manuel Miranda and Antonio Suárez[17]

The competitive Lotka–Volterra system of ordinary differential equations has been intensively studied in recent years. In the present poster we will focus on this system with a higher-order term added

$$\begin{cases} u' = u(b_1 - u - a_{12}v) \\ v' = v(b_2 - v - a_{21}u - huv) \end{cases} \tag{3}$$

where $b_i \in \mathbb{R}$, $a_{ij} > 0$ and $h \geq 0$. Observe that in the classical Lotka–Volterra model, $h = 0$, the competition of the species v is described by a linear term $-a_{21}u$. However, recent works (see [3], [4] and references therein) describe observed species with non–linear effects of competition. It is well-known that when $h = 0$ then if

$$(b_1, b_2) \in \mathcal{R} := \{(b_1, b_2) \in \mathbb{R}^2_+ : a_{12}^{-1}b_1 > b_2 > a_{21}b_1\}$$

then there exists a unique coexistence state $(u^*, v^*) > (0, 0)$ globally stable, hence $\mathcal{R}$ is the maximal biodiversity region. We will ask if the inclusion of this new nonlinear term changes the structure of the maximum biodiversity domain. We will see that when h increases, then the region of maximum biodiversity increases. Specifically, in the case $h > 0$, we prove the existence of a region $\mathcal{R}_h$ with $\mathcal{R} \subset \mathcal{R}_h$, such that, if $(b_1, b_2) \in \mathcal{R}_h$ then there exists at least a locally asymptotically stable coexistence state. Finally, we also show that if $(b_1, b_2) \in \mathcal{R}_h \setminus \mathcal{R}$ two saturated equilibria exist: one is stable and the other is a saddle point. In this subregion, we demonstrate the existence of a separatrix curve, as is the case for $h = 0$ (see [2]), which separates the basin of attraction of the stable saturated steady state, which is always located below the curve, and the basin of attraction of the semi-trivial state located above the curve, which always exists in this case. This result is an extension of [1] for $h > 0$.

References

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[17] Departamento de Ecuaciones Diferenciales y Análisis Numérico, Universidad de Sevilla