Continuum of solutions from a continuation theorem on open sets

Vinicius Kobayashi Ramos, Carlos Alberto Santos and Willian Cintra[18]

In this poster we will present a result that provides the existence of a continuum of positive solutions (λ, u) of $u = K(\lambda, u)$, emanating from a point (λ_0, u_0) with non zero Leray Schauder Index, where K is a compact operator defined on $\overline{\mathcal{U}}$, $\mathcal{U}$ is an open subset of $\mathbb{R} \times E$ (E Banach space) and u_0 is an isolated solution of $u = K(\lambda_0, u)$. The result is an improvement of Theorem 2.2 of [1] which requires the set of solutions for $\lambda = \lambda_0$ to be unitary and $\mathcal{U} = \mathbb{R} \times E$. By applying the result for $\lambda_0 = 0$ and an appropriated $\mathcal{U}$, we prove that the problem

$$\begin{cases} -\Delta u - \lambda u \Delta(u^2) = \mu u - u^p & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with $\mu > \lambda_1$ and $p > 1$, admits a positive solution for each $\lambda > -1/\left(2\mu^{\frac{2}{p-1}}\right)$. Also we prove some existence and qualitative information about positive solutions of a Kirchhoff-Carrier-type problem.

References

[1] Arcoya, D., De Coster, C., Jeanjean, L. and Tanaka, K., Continuum of solutions for an elliptic problem with critical growth in the gradient. *Journal Of Functional Analysis*, 268 (8), 2298–2335 (2015).

[2] Fernández-Rincón, S. and López-Gómez, J., The singular perturbation problem for a class of generalized logistic equations under non–classical mixed boundary conditions. *Advanced Nonlinear Studies*, 19 (1), 1–27 (2019).

[3] Amann, H. and Crandall, M. G., On some existence theorems for semi–linear elliptic equations. *Indiana University Mathematics Journal*, 27 (5), 779–790 (1978).

[4] Rabinowitz, P. A., Global theorem for nonlinear eigenvalue problems and applications. *Contributions To Nonlinear Functional Analysis*, 11–36 (1971).

[18]Departamento de Matemática, Universidade de Brasília