Resúmenes de la Conferencia Internacional sobre Álgebra Conmutativa, Combinatoria y Computacional en memoria de Pilar Pisón Casares

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# ICCCCA in memory of Pilar Pisón Casares 

Sevilla, February 11-16, 2008

Edited by:
Francisco-Jesús Castro-Jiménez
Alberto Vigneron-Tenorio

# ICCCCA in memory of Pilar Pisón Casares 

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## Schedule

Monday, 11th February<br>08:30-09:30 Registration,opening and welcome<br>09:30-10:30 B. Sturmfels, Toric Dynamical Systems<br>10:30-11:00 Coffee break<br>11:00-12:00 W. Bruns, Experiments in lattice polytopes<br>12:00-13:00 M. Chardin, The equation(s) of the image of a rational map<br>13:00 Lunch<br>15:30-16:30 A. Bravo, Hypersurface singularities in positive characteristic and stratification of the singular locus<br>16:30-17:30 M. Boij, Graded Betti numbers of Cohen-Macaulay modules up to multiplication by scalars<br>17:30-18:00 Coffee break<br>18:00 Poster Session<br>Tuesday, 12th February<br>09:30-10:30 G.-M. Greuel, Invariants and Classification of Hypersurface Singularities in Positive Characteristic<br>10:30-11:00 Coffee break<br>11:00-12:00 A. Simis, Cremona maps by way of ideals of linear type and linear syzygies<br>12:00-13:00 S. Zarzuela Modules, ideals and their Rees algebras<br>13:00 Lunch<br>15:30-16:30 D. Maclagan, Smooth multigraded Hilbert schemes<br>16:30-17:30 I. Ojeda, On the combinatorics of the free resolutions of semigroup algebras<br>17:30-18:00 Coffee break<br>18:00 Poster Session<br>19:00 Visit to Reales Alcázares<br>Wednesday, 13th February<br>09:30-10:30 D. Eisenbud, Betti Numbers of Graded Modules and Cohomology of Vector<br>Bundles<br>10:30-11:00 Coffee break<br>11:00-12:00 J.C. Migliore, Gorenstein Hilbert functions<br>12:00-13:00 K. Altmann, Projective varieties with torus action<br>13:00 Lunch<br>15:30-16:30 M. Casanellas, Algebraic models of evolution<br>16:30-17:30 D. Cutkosky, Semigroups of valuations<br>17:30-18:00 Coffee break<br>18:00 Poster Session

## Thursday, 14th February

08:30-09:30 J. Herzog, How to compute the Stanley depth of a monomial ideal
09:30-10:30 S. Gusein-Zade, Power structure over the Grothendieck ring of complex quasi-
projective varieties and invariants of configuration spaces.
11:00 Tourist Visit to Osuna
Friday, 15th February
09:30-10:30 B. Teissier, Overweight deformations of toric varieties
10:30-11:00 Coffee break
11:00-12:00 T. Hibi, Gotzmann ideals of the polynomial ring
12:00-13:00 I. Itenberg, Recursive formulas for Welschinger invariants
13:00 Lunch
15:30-16:30 E. Gorla, Linkage of schemes defined by minors and pfaffians
16:30-17:30 T. Markwig, The $j$-Invariant of a Tropical Elliptic Curve
17:30-18:00 Coffee break
18:00 Poster Session
19:00 Visit to the Cathedral
Saturday, 16th February
09:30-10:30 A. Dickenstein, $A$-discriminants
10:30-11:30 A. Campillo, Toric Geometry and Poincaré Series
12:00 Closing ceremony homage to Pilar Pisón Casares

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## Conference Abstracts

# Projective varieties with torus action 

Klaus Altmann<br>Freie Universität Berlin


#### Abstract

Projective toric varieties are described either by lattice polytopes in the character group of the torus, or by a polyhedral fan. In the latter case, the projective embedding is encoded by a piecewise linear function on the fan. We will generalize this concept to the case of torus actions of smaller dimension such as the action of $(C *)^{n}$ on $\operatorname{Grass}(2, n)$. The resulting description includes a direct information about the position of the orbits inside the projective variety. This is joint work with Georg Hein, Essen.


# Graded Betti numbers of Cohen-Macaulay modules up to multiplication by scalars 

Mats Boij<br>KTH Stockholm


#### Abstract

In a joint work together with Jonas Söderberg on the Multiplicity Conjecture of Herzog, Huneke and Srinivasan we studied the set of possible Betti diagrams of graded Cohen-Macaulay modules up to multiplication by rational numbers. This lead us to formulate conjectures on the structure of this set as a simplicial convex polytope generated by the pure Betti diagrams, where the Multiplicity Conjecture is an immediate consequence.

Recently, all of these conjectures have been proved in characteristic zero. First, the existence of modules with pure resolutions was established by Eisenbud, Floystad and Weyman, and now the remaining parts were proved by Eisenbud and Schreyer. I will give the background of the conjectures and present several different consequences of them


# Hypersurface singularities in positive characteristic and stratification of the singular locus 

Ana Bravo<br>Universidad Autónoma de Madrid


#### Abstract

It is a well known fact that resolution of singularities can be achieved as a byproduct of log resolution of ideals. In this work we show that given a hipersurface H over a field K we can define an upper-smicontinuous function $\eta$ that stratifies H in smooth strata. Then, by means of an inductive argument and a form of elimination, we show that after a finite sequence of monoidal transformations with centers the maximun stratum of $\eta$, the ideal of the total transform of H has a "monomial form in less variables", in a sense that will be made precise in the talk. Over fields of characteristic zero this procedure gives rise to an already known algorithm of resolution of singularities.


# Experiments in lattice polytopes 

Winfried Bruns<br>Universität Osnabrück

We will discuss an experimental approach to various questions concerning lattice polytopes, affine monoids and the corresponding toric varieties. The questions considered so far are
(i) the covering of normal affine monoids by unimodular and simplicial submonoids,
(ii) the construction of lattice polytopes that represent very ample, but not arithmetically normal Cartier divisors on projective toric varieties,
(iii) the existence of smooth projective varieties that have an embedding that is not arithmetically normal or not defined by degree 2 binomials.

The attacks on (i) and (ii) have been successful, resulting in counterexamples to conjectures (and even to arXived preprints). Examples with the properties searched for in (iii) have not yet been found, but it may nevertheless be interesting to discuss some related algorithms and their limitations.

This is a joint project with J. Gubeladze (San Francisco).

# Toric Geometry and Poincaré Series 

Antonio Campillo<br>Universidad de Valladolid


#### Abstract

Poincaré series associated to multi-index filtrations had been developed in a geometric-topological context in joint work with F. Delgado and S. Gusein-Zade, and by other authors. For more than one index, even the definition (nodays a natural integral) is relatively new in the literature, the first time it appears is in a paper with F. Delgado and K.H. Kiyek in 1994 as a method for simplifying information on curve singularities. Often filtrations are valuative ones, providing geometric meaning to Poincaré series. Thus, for affine toric and for some pretoric varieties, respective results by A.Lemahieu and by P. González-Pérez and F. Hernando, show that such geometrical Poincaré series can be directly obtained from the algebraic Hilbert function of the multi-index natural grading. From it, we show how, in the toric case, the results of P. Pisón-Casares on the minimal and short resolutions can be fully applied to describe Poincaré series in the geometrical context, including motivic Poincaré series. A natural question is how long toric type mathematics, as those considered by Pilar and collaborators, can be used for describing Poincaré series in general cases. We approach this question by showing that the obstructions to have a direct description of Poincaré series can be homologically formulated, in such a way that the toric case becomes homologically trivial. It is a work in progress joint with A. Lemahieu.


# The equation(s) of the image of a rational map 

Marc Chardin<br>Université Pierre et Marie Curie


#### Abstract

We will present an approach to computing the equation(s) of the image of a rational map by using syzygies. We will first describe the geometric ideas we use, and then apply them in at least two cases : the implicitization problem (i.e. the case where the image is an hypersurface in a projective space) and the equations of projective rational curves. The first case (implicitization) was studied by many authors in the last years. We will present today's knowledge on this issue, in particular our recent contributions with Laurent Busé and Jean-Pierre Jouanolou. On the second case (the equations of rational curves), we will explain how this method gives an alternative way to recover the bound of L'vovsky on the regularity of rational curves. If some time is left, we will show a natural extension of this result to estimates on the Castelnuovo-Mumford of images of rational maps from a one dimensional scheme to a projective scheme.


# Semigroups of valuations 

Dale Cutkosky<br>University of Missouri


#### Abstract

Suppose that $R$ is a noetherian domain and $v$ is a valuation dominating $R$. The value groups of $v$ are well understood and have been classified, but the semigroups of valuations dominating $R$ are still not well understood. We discuss recent joint work with Bernard Teissier on understanding these semigroups. We give a number of examples showing strange behavior of these semigroups. In Zariski Samuel, two general constraints on value semigroups are given. We give an example of a semigroup satisfying these conditions which is not the semigroup of a valuation dominating a noetherian local ring. This follows from a new bound on the growth of a semigroup of a valuation.


# $A$-discriminants 

## Alicia Dickenstein Universidad de Buenos Aires


#### Abstract

We will address several questions related to dual toric varieties: their tropical versions, the characterization of self-dual toric varieties and an application to finding systems of two bivariate real polynomials with many real zeros. This will summarize joint work with M. Bourel, E.M. Feichther, A. Rittatore, J. M. Rojas, K. Rusek, J. Shih, and B. Sturmfels.


# Betti Numbers of Graded Modules and Cohomology of Vector Bundles 

David Eisenbud<br>Mathematical Sciences Research Institute, Berkeley


#### Abstract

Mats Boij and Jonas Soederberg (math.AC/0611081) conjectured that the Betti table of a Cohen-Macaulay module over a polynomial ring can be decomposed in a certain way as a positive linear combination of Betti tables of modules with pure resolutions. I will talk about the proof of this conjecture in characteristic zero, and some of its consequences. This is joint work with Gunnar Floeystad, Frank-Olaf Schreyer, and Jerzy Weyman.


# Linkage of schemes defined by minors and pfaffians 

Elisa Gorla<br>University of Zürich


#### Abstract

Many varieties which are classically studied in algebraic geometry are defined by determinantal or pfaffian equations. In our talk, we will give an overview of the known results about the linkage class of schemes cut out by minors or pfaffians. This will include some of our recent work on linkage of schemes cut out by minors of matrices with polynomial entries, and on linkage of varieties cut out by pfaffians (the latter is joint work with E. De Negri).


# Invariants and Classification of Hypersurface Singularities in Positive Characteristic 

Gert-Martin Greuel<br>Universität Kaiserslautern


#### Abstract

This is a report about new results on isolated hypersurface singularities in positive characteristic. Most of the well known invariants of such singularities which are classical over the complex numbers, have to be reconsidered and modified over algebraically closed fields in positive charactereistic. This is in particular the case for the Milnor and Tjurina number, the determinacy, the Newton number and various notions of nondegeneracy for semi piecewise-homogeneous singularities. These invaraints are used to classify the Kunimodal singularities in positive characteristic. The results are joint work with Yousra Boubakri and based on her PhD thesis.


# Power structure over the Grothendieck ring of complex quasi-projective varieties and invariants of configuration spaces 

Sabir Gusein-Zade<br>Moscow State University


#### Abstract

Power structure over a ring is a method to give sense to an expression of the form $\left(1+a_{1} t+a_{2} t^{2}+\ldots\right)^{v}$ where $a_{i}$ and m belong to the ring. This notion is closely connected with the notion of a lambda-structure. A (or rather the) power structure over the Grothendieck ring of complex quasi-projective varieties was described by the author with A.Melle and I.Luengo. It gives a new approach to formulae for generating series of invariants of some configuration spaces and to motivic versions of some classical invariants.


# How to compute the Stanley depth of a monomial ideal 

Juergen Herzog<br>Universität Duisburg-Essen


#### Abstract

This is the report on joint work with Marius Vladoiu and Xinxian Zheng. Let $K$ be a field, $S=K\left[x_{1}, \ldots, x_{n}\right]$ the polynomial ring in $n$ variables, and $M$ a finitely generated $\mathbb{Z}^{n}$-graded $S$-module. Let $u \in M$ be a homogeneous element in $M$ and $Z$ a subset of $\left\{x_{1}, \ldots, x_{n}\right\}$. We denote by $u K[Z]$ the $K$-subspace of $M$ generated by all elements $u v$ where $v$ is a monomial in $K[Z]$. The $\mathbb{Z}^{n}$-graded $K$-subspace $u K[Z] \subset M$ is called a Stanley space of dimension $|Z|$, if $u K[Z]$ is a free $K[Z]$-module. A Stanley decomposition of $M$ is a presentation of the $\mathbb{Z}^{n}$-graded $K$-vector space $M$ as a finite direct sum of Stanley spaces $\mathcal{D}: M=\oplus_{i=1}^{m} u_{i} K\left[Z_{i}\right]$ in the category of $\mathbb{Z}^{n}$-graded $K$-vector spaces. The number


$$
\text { sdepth } \mathcal{D}=\min \left\{\left|Z_{i}\right|: i=1, \ldots, m\right\}
$$

is called the Stanley depth of $\mathcal{D}$. The Stanley depth sdepth $M$ of $M$ is defined to be the maximum of all the numbers sdepth $\mathcal{D}$ where $\mathcal{D}$ runs over all possible Stanley decompositions of $M$.

It is conjectured by Stanley that depth $M \leq$ sdepth $M$ for all $\mathbb{Z}^{n}$-graded $S$-modules $M$. The conjecture is widely open.

A priori it is not clear how to compute sdepth $M$. We will discuss this question for $\mathbb{Z}$-graded modules of the form $I / J$ where $J \subset I \subset S$ are two monomial ideals. We fix an integer vector $g \in \mathbb{Z}^{n}$ with the property $a \leq g$ for all $a \in \mathbb{Z}^{n}$ with $x^{a} \in I \backslash J$ and define the characteristic poset $P_{I / J}^{g}$ of $I / J$ with respect to $g$ as

$$
P_{I / J}^{g}=\left\{a \in \mathbb{Z}^{n}: x^{a} \in I \backslash J, \quad a \leq g\right\}
$$

with the partial order given by componentwise comparison. One of the main results will be that the Stanley depth of $I / J$ can be computed by considering the (finitely many) partitions of $P_{I / J}^{g}$ as a disjoint union of intervals.

Similarly one defines the fdepth of a module in terms of prime filtrations. One always has fdepth $M \leq$ depth $M$, sdepth $M$. Again it can be shown that fdepth $M$ can be computed in a finite number of steps. We give several applications, and show for example that each ideal of Borel type (ideal of nested type) satisfies Stanley's conjecture.

# Gotzmann ideals of the polynomial ring 

Takayuki Hibi<br>University of Osaka


#### Abstract

Let $A$ be the polynomial ring in $n$ variables over a field. A Gotzmann ideal of $A$ is a homogeneous ideal of $A$ whose graded Betti numbers coincide with those of its lexsegment ideal. Our goal is to classify all Gotzmann ideals of $A$ generated by at most $n$ homogeneous polynomials. This is a joint work with Satoshi Murai, to appear in Math. Z.


# Recursive formulas for Welschinger invariants 

Ilia Itenberg<br>Université Louis Pasteur


#### Abstract

Welschinger invariants are designed to bound from below the number of real rational curves which pass through a given real generic collection of points on a real rational surface. In some cases these invariants can be calculated using Mikhalkin's approach which deals with a corresponding count of tropical curves.

Using this approach and tropical Caporaso-Harris type formulas, we establish a logarithmic equivalence of Welschinger and Gromov-Witten invariants in several situations.


# Smooth multigraded Hilbert schemes 

Diane Maclagan<br>University of Warwick


#### Abstract

Given a homomorphism $\operatorname{deg}: Z^{n} \rightarrow A$, where $A$ is an abelian group, we grade $S:=k\left[x_{1}, \ldots, x_{n}\right]$ by $\operatorname{deg}\left(x_{i}\right)=\operatorname{deg}\left(e_{i}\right)$. The multigraded Hilbert scheme $H_{S}^{h}$ of Haiman and Sturmfels associated to a function $h: A \rightarrow N$ parameterizes all ideals $I$ subset $S$ with multigraded Hilbert function $h$. I will discuss joint work with Greg Smith showing that $H_{S}^{h}$ is smooth and irreducible when $n=2$, so $S=k\left[x_{1}, x_{2}\right]$.


# The $j$-Invariant of a Tropical Elliptic Curve 

Thomas Markwig<br>Technische Universität Kaiserslautern


#### Abstract

It turns out that many interesting properties of algebraic varieties are preserved under the process of tropicalisation. For the purpose of this talk we suppose that our algebraic varieties are defined over the field of Puiseux series and that the tropicalisation map assigns to a Puiseux series its order. If we start on the algebraic side with a smooth plane cubic, i.e. a curve of genus one, then under good circumstances the tropicalisation will be a piecewise linear graph of genus one, i.e. the graph will have precisely one loop. It turns out that the length of this loop is related to the $j$-invariant of the cubic. In particular that the presence of the loop generically only depends on the $j$-invariant.


# Gorenstein Hilbert functions 

Juan C. Migliore<br>University of Notre Dame. Indiana


#### Abstract

What are the possible Hilbert functions of graded Artinian Gorenstein algebras? Many authors have studied this question. When the algebras have the Weak Lefschetz Property (WLP), the Hilbert functions are the so-called Stanley-Iarrobino (SI) sequences. This implies, in particular, that they are unimodal. In codimension 3, the possible Hilbert functions are again precisely the SI-sequences, even though it is still an open question whether all such algebras have WLP. In codimension five or more, it is known that non-unimodal examples exist. We will present some asymptotic results describing "how non-unimodal" they can be, and in the process answer an old conjecture of Richard Stanley. This leaves codimension 4 hanging in the balance. It is not known whether non-unimodal, or even non-SI, examples exist, although it is known that not all such algebras have WLP. We will present recent work showing that at least for low initial degree, all occurring Hilbert functions are SI-sequences (hence unimodal). All new work described is joint with Uwe Nagel and Fabrizio Zanello.


# On the combinatorics of the free resolutions of semigroup algebras 

Ignacio Ojeda<br>Universidad de Extremadura


#### Abstract

Let $S$ be a finitely generated, combinatorially finite, cancellative and commutative semigroup and let $k[S]$ be its associated $k$-algebra (where $k$ denotes an arbitrary field). If $L=\left\{n_{1}, \ldots, n_{r}\right\}$ is a (fixed) generating set for $S$, it is well-known that $k[S]$ has a natural ( $A:=k\left[x_{1}, \ldots, x_{r}\right]$ )-module structure. So, one can study $k[S]$ via its minimal free resolution of $k[S]$ as $A$-module. On the other hand, for any partition $\{E, A\}$ of L , one may define a ( $B:=k\left[x_{1}, \ldots, x_{f}\right]$ )-module structure on $k[S]$, provided that $E=\left\{n_{1}, \ldots, n_{f}\right\}$; moreover, if the positive spans of $E$ and $L$ agree, $k[S]$ is a finite generated $B$-module. In this case, the minimal free resolution of $k[S]$ as $B$-module, the so-called the short resolution of $k[S]$ by P. Pisón who in 2003 discovered its properties, also gives useful information about the semigroup algebra of $S$.

This talk will be mainly devoted to the works of P. Pisón and her collaborators about the combinatorial descriptions for both resolutions and the Pisón-Vigneron algorithm for their computation. New results on the combinatorial description of the minimal free resolutions of $\mathrm{k}[\mathrm{S}]$ will be pointed out; these last are joint work with A. Vigneron.


# The defining ideals of conjugacy classes of nilpotent matrices and a conjecture of Weyman 

Mercedes Helena Rosas<br>Universidad de Sevilla<br>join work with Riccardo Biagioli (Lyon) and Sara Faridi (Halifax)


#### Abstract

Tanisaki introduced generating sets for the defining ideals of the schematic intersections of the closure of conjugacy classes of nilpotent matrices with the set of diagonal matrices. These ideals are naturally labeled by integer partitions. Given such a partition $\lambda$, we define several methods to produce a reduced generating set for the associated ideal $I_{\lambda}$.

For particular shapes we find nice generating sets. By comparing our sets with some generating sets of $I_{\lambda}$ arising from a work of Weyman, we find a counterexample to a longstanding conjecture of Weyman.


# Cremona maps by way of ideals of linear type and linear syzygies 

Aron Simis<br>Universidade de Pernambuco


#### Abstract

This is based on joint work with F. Russo and is still a bit experimental. The idea of looking at the syzygies of the forms defining a rational map from projective $n$-space space to projective $m$-space goes back to the work of Hulek-Katz-Schreyer, in the hope of getting criteria for the map to be birational (at least in the case where $m=n$ ). In this talk I will give other recent results, sometimes more encompassing, sometimes chacteristic free. I will mention related work by Ciliberto et. al. and Eisenbud et. al.


# Toric Dynamical Systems 

## Bernd Sturmfels <br> University of California at Berkeley


#### Abstract

Toric dynamical systems are known as complex balancing mass action systems in the mathematical chemistry literature, where many of their remarkable properties have been established. They include as special cases all deficiency zero systems and all detailed balancing systems. One feature is that the steady state locus of a toric dynamical system is a toric variety, which has a unique point within each invariant polyhedron. We develop the basic theory of toric dynamical systems in the context of computational algebraic geometry and show that the associated moduli space is also a toric variety. It is conjectured that the complex balancing state is a global attractor. We prove this for detailed balancing systems whose invariant polyhedron is two-dimensional and bounded. This is joint work with Gheorghe Craciun, Alicia Dickenstein and Anne Shiu (arXiv:0708.3431).


# Overweight deformations of toric varieties 

Bernard Teissier<br>Université Paris VII


#### Abstract

Given an affine toric variety $X$ described by binomials in some system of coordinate, and a monomial order on the corresponding polynomial ring, some particular deformations of the binomial equations, called "overweight" with respect to the order, produce a family $X_{t}$ with the property that the monomial order induces a valuation on the ring of $X_{t}$ and some toric resolutions of the singularities of $X=X_{0}$ deform to a resolution of $X_{t}$ near the center of the valuation.


# Modules, ideals and their Rees algebras 

Santiago Zarzuela<br>Universidad de Barcelona


#### Abstract

In studying Rees algebras of modules and related concepts (reductions, integral closures...) one may attach to a module various ideals. For a particular purpose one can then reduce to the study of the case of ideals which is easy to handle. In this talk we shall review some of these ideals, in particular the "Bourbaki ideal of a module" introduced by A. Simis, B. Ulrich and W. V. Vasconcelos.

This is based on a joint work with Ana Luisa Branco-Correia.


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## List of Posters

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Complete intersections in certain affine monomial curves
I. Bermejo and I. García-Marco

Groebner bases for submodules of $\mathbb{Z}^{n}$
G. Boffi and A. Logar

The implicitization problem for the case $\phi: \mathbb{P}^{n} \rightarrow\left(\mathbb{P}^{1}\right)^{n+1}$ N. S. Botbol

Short rational generating functions for multiobjective linear integer programming
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