

C1: Theoretical background and basic results

Elliptic PDEs: A. Suárez

1. Short bio:

Antonio Suárez is currently a Full Professor in the Department of Differential Equations and Numerical Analysis. He received his BSc in Mathematics from the University of Seville in 1994 and his PhD in Mathematics from the same university in 1999. His doctoral thesis focused on PDE models arising from population dynamics.

His research has primarily focused on the theoretical analysis of nonlinear partial differential equations (PDEs), particularly elliptic problems. Most of these models originate from questions in population dynamics or tumour dynamics. Regarding this topic, my research focuses on two variants that make the models more realistic. On the one hand, models in which diffusion (spatial movement of the species) is non-linear, i.e. it is not a random movement but depends on the density of the species itself. And on the other hand, the inclusion of non-local terms, which take into account global aspects of the species, not just local ones. which allow for more realistic descriptions of biological interactions.

Over the past 15 years, he has served as principal investigator of competitive national and international research projects. He has also supervised 10 PhD theses, some of them as part of co-tutelle agreements with different universities in Brazil.

2. A short abstract of the course:

2.1. Lecture 1

In the first part of the course, we introduce the motivation behind the study of elliptic PDEs. We will present the different notions of solution as well as the spaces in which such solutions exist. This section will be devoted to the study of:

1. **Linear elliptic equations.** We will investigate the existence, uniqueness, and regularity of solutions to linear elliptic equations, with particular emphasis on the functional framework and boundary conditions under which these problems are well posed.
2. **Maximum principles.** We will study maximum and comparison principles for elliptic equations and their applications to the qualitative analysis of solutions, including uniqueness, positivity, and comparison results.

3. **Eigenvalue problems.** We will consider eigenvalue problems associated with elliptic operators, focusing on the existence and properties of eigenvalues and eigenfunctions, as well as their role in understanding the qualitative behavior of solutions.

2.2. Lecture 2

In the second part of the course, we will focus on nonlinear problems and on the mathematical techniques used to establish the existence, uniqueness, and qualitative properties of their solutions. In particular, we will illustrate the applicability of several complementary approaches:

1. **Monotonicity methods.** We will study the method of sub- and supersolutions as a powerful tool for proving the existence of solutions to nonlinear problems. We will discuss the construction of suitable ordered sub- and supersolutions and their applications to boundary value problems.
2. **Bifurcation methods.** We will introduce bifurcation techniques to analyze the existence and structure of branches of nontrivial solutions depending on a parameter. Particular attention will be paid to the emergence of new solutions from known solutions and to the qualitative changes that may occur as parameters vary.
3. **Applications to population dynamics.** Finally, we will present applications of these methods to different mathematical models arising in population dynamics. These examples will illustrate how nonlinear differential equations can be used to describe phenomena such as population growth, competition, coexistence, and spatial interactions between species.