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The initial-to-final-state inverse problem for the Schrödinger equation

Short bio:

A short abstract of the course:

The initial-to-final-state inverse problem for the Schrödinger equation arises naturally as a framework for studying the theoretical aspects of data-driven prediction in quantum mechanics. This problem aims to approximate the solution of the initial-value problem for the Schrödinger equation without knowing the exact Hamiltonian, that is, without knowing the exact coefficients of the equation. Instead, one has access to the initial and final states of a large set of solutions, and uses this data to approximate the solution to the problem at hand.

The main difficulty comes from not knowing the exact Hamiltonian that governs the dynamics. To build a theoretical framework for this problem, Alberto Ruiz and I formulated an inverse problem: determine the Hamiltonian of a quantum system—the coefficients of the Schrödinger equation—from knowledge of the final state corresponding to each initial state. In this lecture, we will focus on the simplest case, where the Hamiltonian is given by $-\Delta + V$ and the electric potential V is not compactly supported (V does not necessarily vanish outside a bounded region). I will present several uniqueness results for the inverse problem, both for time-dependent potentials $V = V(t, x)$ and for stationary potentials $V = V(x)$. These results explain why the electric potentials are determined by the corresponding initial and final states of all solutions. As a consequence, one expects to be able to solve the data-driven prediction problem in quantum mechanics.

The theorems I will discuss come from collaborations with Alberto Ruiz, Manuel Cañizares, Ioannis Parisis, and Athanasios Zacharopoulos.