Null controllability and stabilisation of degenerate wave equations

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Controllability and stabilisation problems for wave equations have attracted considerable attention in recent years, leading to significant developments in the analysis of various classes of evolution equations. In particular, starting from the classical theory of non-degenerate wave equations, several results have been established for more general models, including equations exhibiting degeneracies in their principal coefficients. Such degeneracies introduce substantial mathematical difficulties, as they may affect the well-posedness of the associated problems, the natural functional framework in which solutions should be considered, and the validity of classical controllability and stabilisation results.

The first part of this talk will focus on the null controllability of degenerate wave equations. Given an initial state, the system is said to be *null controllable at time* $T > 0$ if there exists a suitable control such that the corresponding solution u satisfies $u(T) \equiv u_t(T) = 0$ throughout the spatial domain. In other words, the control drives the system to rest at a prescribed final time, regardless of its initial configuration.

In the presence of degeneracy, classical null controllability results for non-degenerate wave equations cannot, in general, be directly applied. The degeneracy may lead to a loss of coercivity in the associated energy estimates and requires a careful choice of the functional setting. To overcome these difficulties, we will develop an approach based on the energy method and the introduction of suitable Hilbert spaces adapted to the structure of the degenerate operator. Within this framework, we will establish global null controllability results, identifying conditions under which the control can drive the solution to zero in finite time.

The second part of the talk will address the stabilisation problem for degenerate wave equations through a boundary damping. Unlike null controllability, which aims to steer the system exactly to rest at a prescribed finite time, stabilisation concerns the long-time behaviour of solutions. More precisely, we will investigate whether an appropriate boundary feedback mechanism can dissipate the energy of the system and ensure that solutions converge to equilibrium as time tends to infinity. The analysis will focus on the role of the degeneracy in the energy dissipation mechanism and on the conditions required to obtain suitable stabilisation properties.

This talk contribute to the understanding of control and stabilisation mechanisms for degenerate hyperbolic equations and illustrate how energy estimates and appropriately chosen functional spaces can be used to address problems for which standard techniques may not be directly applicable.

Short biography:

Genni Fragnelli is Associate Professor at the University of Siena since October 2024. She obtained her PhD in February 2002 at the University of Tuebingen under the supervision of Professor R. Nagel. She was a PostDoc at the University of Roma Tor Vergata (2002-2005), at the University of Siena (2005-2010); then she was researcher at theUniversity of Bari Aldo Moro (2010-2017) and Associate Professor at the same University until 2021 and from 2021 to 2024 at the University of Tuscia. Her research activity is mainly focused on Control Theory, Semigroup Theory, Nonlinear Analysis, Populations Models and Topological Degree.