Models and methods for pricing renewable energy certificates and some derivatives

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Outline

1. Introduction and objectives
2. Renewable Energy Certificates (RECs)
   - Mathematical model
   - Numerical method
   - Numerical results
3. REC derivatives
   - REC derivatives: options and futures
   - Mathematical model
   - Existence of a solution for the PDE pricing model
   - Numerical method
   - Numerical results
4. Jumps and PIDE model for RECs prize
5. Conclusions

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Energy markets

- **Energy consumption** has been **growing** and this trend will continue.
- **Policy commitments** ongoing, such as renewable energy or efficiency targets.
- The **average annual growth rate in energy demand** is estimated to **grow** in the next years.
- The **energy growth** mainly depends on how the energy is used across industry, transport and buildings.
- **Climate issues** (global warming, strange weather patterns, etc) clearly affect energy resources and, therefore, energy markets.
- **Recent geopolitical issues** affect energy markets and prices.

Energy markets

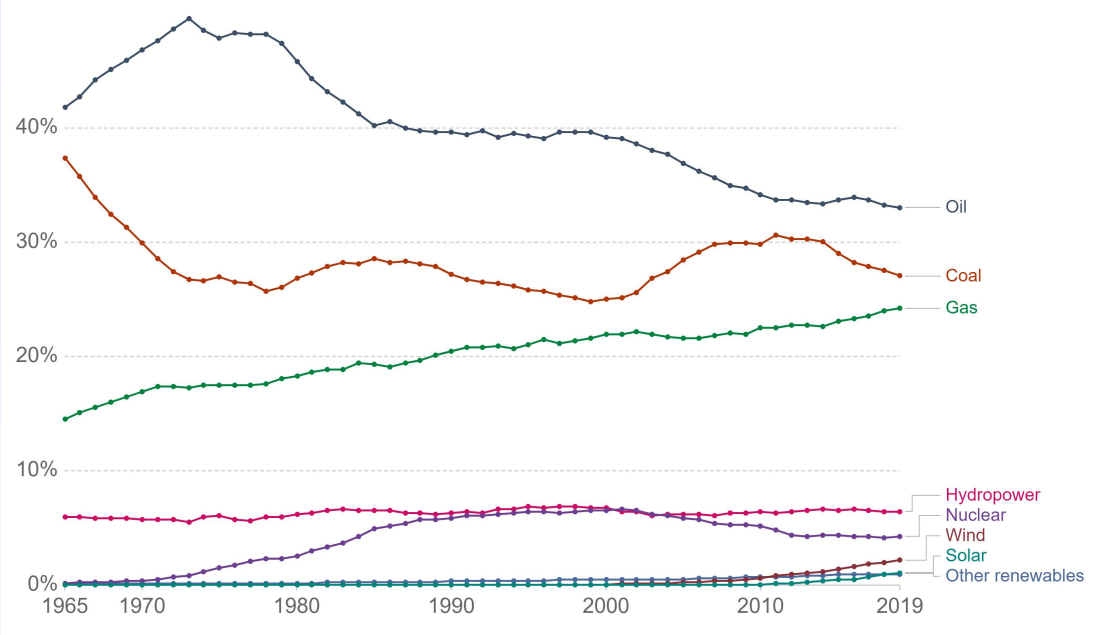


Figure: World share of energy consumption by source.

(*Source*: BP Statistical Review of World Energy (2020)).

The electricity market

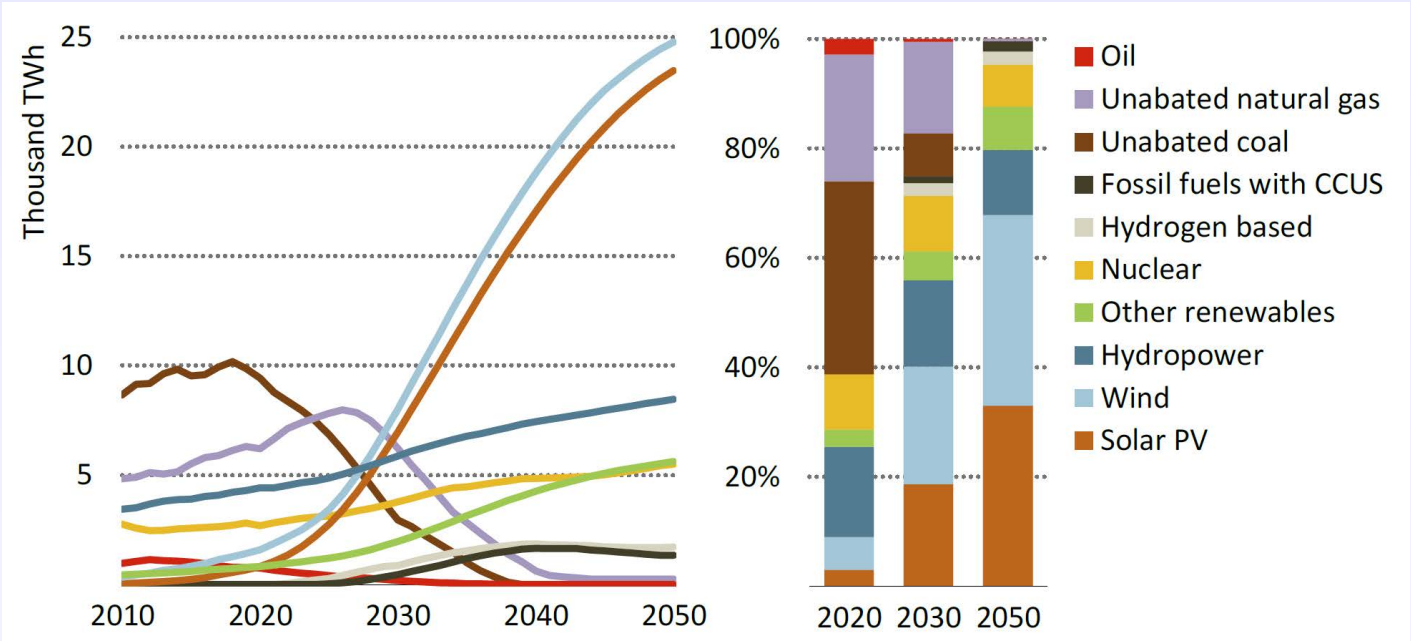


Figure: Global electricity generation by source.

(*Source*: International Energy Agency (IEA), 2021).

Support schemes for renewable energy

Most governments have recognised the **benefits of renewable energies** and have established **targets** for them:

- to produce energy for final consumption,
- to increase their weigth in electricity generation,
- to rise the photovoltaic generation capacity.

Renewable energy technologies often require a large investment, hence alternative market tools are required to achieve these targets, such as **renewable energy certificates**.

Renewable Energy Certificates (RECs)

Some main ideas on **RECs**:

- The renewable energy generators receive one certificate (**REC**) per specific unit (1 MWh of renewable electricity produced).
- RECs can be sold to a **Load Serving Entity (LSE)**.
- If the quota obligation is not met by LSE, a non-compliance penalty applies (**Alternative Compliance Payment (ACP)**).
- Many countries have adopted **Renewable Portfolio Standards (RPSs)** and trading of RECs.
- RECs are more recent than **cap-and-trade schemes for carbon emissions**.

Some references (non exahustive list)

- Chen, Hobss,Leyffer and Mumson (2006), Tietenberg (2006),
- Carmona, Fehr, Hinz (2009), Carmona, Fehr, Hinz (2010), Carmona, Hinz (2011),
- Tanaka, Chen (2012), Carmona, Delarue, Spinosa, Touzi (2013), Howison, Schwarz (2015),
- **Coulon, Khazaei, Powell (2015), Khazaei, Coulon, Powell (2017),**
- **Hustveit, Frognen, Fleten (2017),**
- **Shrivats, Jaimugal (2020), Shrivats, Firoozi, Jaimugal (2022), Firoozi, Shrivats, Jaimugal (2022),**
- ...

Objectives

- To **model the price of a** renewable energy certificate (**REC**).
- To obtain the PDE which models the REC pricing problem (**non-linear PDE**) and to propose **suitable numerical methods**.
- To introduce **models to determine** the *fair value* of a **derivative** whose underlying is a REC.
- To characterize the price of a REC derivative as a **solution of a linear PDE problem**.
- To propose **suitable numerical methods**.

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Underlying stochastic factors

We assume that the price of a renewable energy certificate is given at time t by a stochastic process $(\boldsymbol{P_t})$ depending on two stochastic factors:

- $\boldsymbol{G_t}$, the renewable generation rate, with the following dynamics:

$$G_t = \exp(\widetilde{G}_t),$$

where $\widetilde{\boldsymbol{G}}_{\boldsymbol{t}}$ is a **Ornstein-Uhlenbeck (OU) process**, which satisfies the following stochastic differential equation (SDE):

Underlying stochastic factors

$$d\widetilde{G}_t = \alpha_g \left(f(t) + \frac{\beta_g}{\alpha_g} P_t - \widetilde{G}_t \right) dt + \sigma_g dW_t^0, \qquad \widetilde{G}_{t_0} = g_0,$$

- α_g is the mean reversion speed of the process,
- $f(t)$ is the seasonality function,
- β_g is the parameter which controls the level of feedback from the price of the certificate,
- σ_g is the volatility,
- W_t^0 is an $\mathcal{F}_t$-adapted $\mathbb{Q}$-Brownian motion.

Underlying stochastic factors

- $\boldsymbol{B_t}$, the number of accumulated green certificates, satisfies the following equation:

$$dB_t = G_t dt, \qquad B_{t_0} = 0.$$

Since the process $\boldsymbol{B_t}$ represents an accumulated quantity, it turns out to be positive and non-decreasing.

We assume the existence of a function P, such that $P_t = P(t, B_t, G_t)$. Once the function P is obtained as the solution of the PDE problem, we can compute the value of P_t for given values of t, G_t and B_t.

Single period case

The price of the renewable energy certificate can be represented as an Itô Integral with respect to W_t^0 as follows

$$dP_t = rP_t dt + Z_t^0 dW_t^0, \quad \text{for } t \in [T-\gamma, T],$$

for some $\mathcal{F}_t$-adapted square integrable process Z_t^0, where

- $\boldsymbol{T}$ is the maturity of the certificate.
- γ is the number of life years of the certificate, so that the life of the certificate starts at $t_0 = T - \gamma$. In the single period case, we take $\gamma = 1$.
- The value of the REC at maturity, $\boldsymbol{P_T}$, is given by the terminal condition:

$$P_T = \psi(B_T),$$

where $\psi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a bounded measurable and decreasing function.

Single period case

The price process of a renewable energy certificate $\boldsymbol{P_t}$ in a single period can be described by the solution of the following FBSDE in the time interval $[t_0, T]$:

$$\begin{cases} d\widetilde{G}_t = \mu_g(t, \widetilde{G}_t, P_t)dt + \sigma_g dW_t^0, & \widetilde{G}_{t_0} = g_0, \\ dB_t = \exp(\widetilde{G}_t)dt, & B_{t_0} = 0, \\ dP_t = rP_t dt + Z_t^0 dW_t^0, & P_T = \psi(B_T). \end{cases}$$

Analogous to Howison, Schwarz (2015), for carbon emissions. However, existence and uniqueness of solution for the FBSDE is an open problem.

Single period case

Using Itô's formula for the process $P_t = P(t, B_t, \widetilde{G}_t)$, the following non-linear PDE for the function $P = P(t, B, \widetilde{G})$ can be obtained:

$$\mathcal{L}_1[P] = \frac{\partial P}{\partial t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 P}{\partial \widetilde{G}^2} + \mu_g(t, \widetilde{G}, P)\frac{\partial P}{\partial \widetilde{G}} + \exp(\widetilde{G})\frac{\partial P}{\partial B} - rP = 0,$$

which defines the final value problem jointly with the condition

$$P(T, B, \widetilde{G}) = \pi_T \mathbb{1}_{[0,R_T)}(B),$$

where $\boldsymbol{\pi_T}$ is the penalty amount at time T and $\boldsymbol{R_T}$ is the requirement (of banked certificates) at time T, and where

$$\mu_g(t, \widetilde{G}, P) = \alpha_g \left(f(t) + \frac{\beta_g}{\alpha_g} P - \widetilde{G} \right),$$

$$f(t) = a_1 \sin(4\pi t) + a_2 \cos(4\pi t) + a_3 \sin(2\pi t) + a_4 \cos(2\pi t).$$

Multiple period case

At each compliance date T^i, $i = 1, \ldots, \gamma - 1$, a jump condition must be applied. Thus, the **value of the certificate at the compliance date** T^i is given by

$$P(T^i, B, \widetilde{G}) = \max\left(\pi_i \mathbb{1}_{[0,R_i)}(B), P\left(T^i_+, \max(0, B - R_i), \widetilde{G}\right)\right),$$

where $\boldsymbol{\pi_i}$ is the penalty amount at time T^i and $\boldsymbol{R_i}$ is the requirement at time T^i.

Main difficulties in the numerical solution

- The drift term depends linearly on the unknown P, so that the first order (convective) term is **non-linear**.
- The PDE is initially posed in a **spatial unbounded domain**.
- The lack of second order derivative with respect to variable B makes the **PDE degenerate**.

Main difficulties in the numerical solution

- The drift term depends linearly on the unknown P, so that the first order (convective) term is **non-linear**.

  Duality method

- The PDE is initially posed in a **spatial unbounded domain**.

  Localization + boundary conditions

- The lack of second order derivative with respect to variable B makes the **PDE degenerate**.

  Higher order Lagrange/Galerkin methods

  Semi-Lagrangian method in one direction

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Semi-Lagrangian method in one direction

M. A. Baamonde-Seoane, M. C. Calvo-Garrido, M. Coulon, C. Vázquez, Numerical solution of a nonlinear PDE model for pricing Renewable Energy Certificates (RECs), *Applied Mathematics and Computation*, 404 (2021), 126199.

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Non-linear convective term

First, we formulate the non-linear PDE in terms of the following non-linear maximal monotone operator:

$$m(P) = \begin{cases} 0, & \text{if } P < 0 \\ P^2, & \text{if } P \geq 0, \end{cases}$$

so that

$$P\frac{\partial P}{\partial \widetilde{G}} = \frac{1}{2}\frac{\partial m(P)}{\partial \widetilde{G}},$$

and the initial PDE can be written in the equivalent form:

$$\frac{\partial P}{\partial t} + \frac{\sigma_g^2}{2}\frac{\partial^2 P}{\partial \widetilde{G}^2} + \alpha_g\left(f(t) - \widetilde{G}\right)\frac{\partial P}{\partial \widetilde{G}} + \frac{\beta_g}{2}\frac{\partial m(P)}{\partial \widetilde{G}} + \exp(\widetilde{G})\frac{\partial P}{\partial B} - rP = 0.$$

Non-linear convective term

Following the Bermúdez-Moreno algorithm, for the constant parameter $\omega > 0$ we introduce the additional unknown

$$\theta = (m - \omega I)(P),$$

where I denotes the identity operator. So, we have

$$\frac{1}{2}\frac{\partial m(P)}{\partial \widetilde{G}} = \frac{1}{2}\frac{\partial \theta}{\partial \widetilde{G}} + \frac{\omega}{2}\frac{\partial P}{\partial \widetilde{G}}.$$

By using the Bermúdez-Moreno lemma, we have the equivalence

$$\theta = m(P) - \omega P \Leftrightarrow \theta = m_\lambda^\omega(P + \lambda\theta),$$

where m_λ^ω denotes the Yosida approximation of the operator $m - I\omega$ with parameter λ.

Non-linear convective term

We impose the relation $2\lambda\omega = 1$ in the choice of the parameters λ and ω so that the Yosida approximation is given by

$$m_\lambda^\omega\left(P+\frac{\theta}{2\omega}\right)=\begin{cases}-\theta-2\omega P, & \text{if } P+\dfrac{\theta}{2\omega}\leq 0,\\[2ex] \theta+2\omega P+\omega^2-\omega\sqrt{4\theta+8\omega P+\omega^2}, & \text{if } P+\dfrac{\theta}{2\omega}\geq 0.\end{cases}$$

The PDE can be rewritten in the equivalent form

$$\mathcal{L}_2[P]=-\frac{\beta_g}{2}\frac{\partial\theta}{\partial\widetilde{G}},\quad \theta=m_\lambda^\omega(P+\lambda\theta),$$

where we have introduced the linear differential operator

$$\mathcal{L}_2[P]=\frac{\partial P}{\partial t}+\frac{\sigma_g^2}{2}\frac{\partial^2 P}{\partial\widetilde{G}^2}+\alpha_g\left(f(t)-\widetilde{G}\right)\frac{\partial P}{\partial\widetilde{G}}+\frac{\beta_g\omega}{2}\frac{\partial P}{\partial\widetilde{G}}+\exp(\widetilde{G})\frac{\partial P}{\partial B}-rP.$$

References

- A. Bermúdez, C. Moreno, Duality methods for solving variational inequalities, *Computers & Mathematics with Applications*, 7 (1981), 43-58.
- I. Arregui, J. J. Cendán, C. Vázquez, A duality method for the compressible Reynolds equation. Application to the simulation of read-write processes in magnetic reading devices, *Journal of Computational and Applied Mathematics*, 175 (2005), 31-40.

Formulation in a bounded domain

▶ O. A. Oleinik, E. V. Radkevic, *Second Order Equations with Nonnegative Characteristic Form*, A.M.S. And Plenum Press, 1973.

- After truncation and scaling in the spatial variables, we consider the **bounded spatial domain** is $\Omega = (0,1) \times (0,1)$, the boundary of which can be decomposed as

$$\Gamma = \bigcup_{i=1}^{2} (\Gamma_i^- \cup \Gamma_i^+)$$

$$\Gamma_i^- = \{(y_1, y_2) \in \Gamma |\, y_i = 0\},\ \Gamma_i^+ = \{(y_1, y_2) \in \Gamma |\, y_i = 1\},\ i = 1, 2.$$

- We introduce the change of time variable $\tau = T - t$, where τ represents the time to maturity T.

Formulation in a bounded domain

The **formulation of the PDE** in divergence form is:

Find $P : [0, \gamma] \times \Omega \to \mathbb{R}$ *such that*

$$\begin{aligned}
\frac{\partial P}{\partial \tau} - Div(\mathcal{A}\nabla P) + v \cdot \nabla P + IP &= \hat{h} + \frac{\beta_g \hat{g}}{2} \frac{\partial \theta}{\partial y_2} \text{ in } (0, \gamma) \times \Omega\,, \\
P(0, \cdot) &= \pi_T \mathbb{1}_{\{\hat{B}\hat{b} < R_T\}} \text{ in } \Omega, \\
\frac{\partial P}{\partial y_1} &= 0 \text{ on } (0, \gamma) \times \Gamma_1^+, \\
\frac{\partial P}{\partial y_2} &= 0 \text{ on } (0, \gamma) \times (\Gamma_2^- \cup \Gamma_2^+),
\end{aligned}$$

jointly with **jump conditions at compliance dates in the multiple period case**.

Formulation in a bounded domain

Moreover, the **diffusion matrix** $\mathcal{A}$, the **velocity field** v, the **linear term** l and the **second member function** $\hat{h}$ have the following expressions:

$$\mathcal{A} = \begin{pmatrix} 0 & 0 \\ 0 & \dfrac{\hat{g}^2\sigma_g^2}{2} \end{pmatrix},$$

$$v = \begin{pmatrix} -\hat{b}\exp(y_2\hat{g} - \bar{g}) \\ -\hat{g}\alpha_g\left(f(T-\tau) - (y_2\hat{g} - \bar{g}) + \dfrac{\beta_g\omega}{2\alpha_g}\right) \end{pmatrix},$$

$$l = r,$$

$$\hat{h} = 0.$$

Characteristics method time discretization

Let us define

$$\frac{DP}{D\tau} = \frac{\partial P}{\partial \tau} + v \cdot \nabla P,$$

which represents the material derivative along the characteristic curve through $y = (y_1, y_2) = (\hat{B}, \hat{G})$ at time $\bar{s}$, $\chi(y, \bar{s}; s)$, which is the solution of the following final value ODE problem:

$$\frac{\partial}{\partial s}\chi(y, \bar{s}; s) = v(\chi(y, \bar{s}; s), s), \quad \chi(y, \bar{s}; \bar{s}) = y.$$

Characteristics method time discretization

- The **characteristics curves** can be exactly computed when $f = 0$ and the components of $\chi^n(y)$ are given by

$$\begin{aligned} \chi_1^n(y) &= \hat{b}\exp(y_2\hat{g} - \bar{g}) + y_1, \\ \chi_2^n(y) &= -\hat{g}\alpha_g\left((y_2\hat{g} - \bar{g}) - \frac{\beta_g\omega}{2\alpha_g}\right) + y_2, \end{aligned}$$

  where $\chi^n(y) = \chi(y, \tau^{n+1}; \tau^n)$.

- We implement a **second order explicit Runge-Kutta scheme** for the cases in which the computation of an analytical solution of the final value problem is not possible.

Some references for numerical techniques

Higher order characteristics:

- A. Bermúdez, M. R. Nogueiras and C. Vázquez, Numerical analysis of convection-diffusion-reaction problems with higher order characteristics finite elements. Part I: Time discretization, *SIAM Journal on Numerical Analysis*, 44 (2006), 1829–1853.

Spatial discretization:

- A. Bermúdez, M. R. Nogueiras and C. Vázquez, Numerical analysis of convection-diffusion-reaction problems with higher order characteristics finite elements. Part II: Fully discretized scheme and quadrature formulas, *SIAM Journal on Numerical Analysis*, 44 (2006), 1854–1876.

Crank-Nicolson characteristics scheme

- We introduce a **time step** $\Delta\tau = \gamma/N_T$ and the **time mesh points** $\tau^n = n\Delta\tau, \quad n = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots, N_T$.
- We approximate the material derivative at time $\tau^{n+\frac{1}{2}}$ by the quotient:

$$\frac{DP}{D\tau} \approx \frac{P^{n+1} - P^n \circ \chi^n}{\Delta\tau},$$

where $\chi^n(y) = \chi(y, \tau^{n+1}; \tau^n)$.

Crank-Nicolson characteristics time discretization

- Crank-Nicolson characteristics time discretization around $(\chi(y, \tau^{n+1}; \tau^{n+\frac{1}{2}}), \tau^{n+\frac{1}{2}})$, $n = 0, \ldots, N_T - 1$.

$$\frac{P^{n+1}(y) - P^n(\chi^n(y))}{\Delta\tau} - \frac{1}{2} Div(\mathcal{A}\nabla P^{n+1})(y) - \frac{1}{2} Div(\mathcal{A}\nabla P^n)(\chi^n(y)) + \frac{1}{2}(I P^{n+1})(y) + \frac{1}{2}(I P^n)(\chi^n(y)) = \frac{1}{2}\hat{h}^{n+1}(y) + \frac{1}{2}\hat{h}^n(\chi^n(y)) + \frac{1}{2}\frac{\beta_g \hat{g}}{2}\left(\frac{\partial\theta^{n+1}}{\partial y_2}(y) + \frac{\partial\theta^n}{\partial y_2}(\chi^n(y))\right).$$

Discretization of PDE

- At each time step, the previous linear PDE is **coupled** with the following **non-linear relation** between P^{n+1} and θ^{n+1}:

$$\theta^{n+1} = m^{\omega}_{1/2\omega}\left(P^{n+1} + \frac{1}{2\omega}\theta^{n+1}\right).$$

- We propose a **fixed point algorithm** in the form:

$$\theta^{n+1,k+1} = m^{\omega}_{1/2\omega}\left(P^{n+1,k+1} + \frac{1}{2\omega}\theta^{n+1,k}\right).$$

Some intermediate calculus: non classical Green formula

Assuming that $\chi^n \in \mathcal{C}^2(\Omega)$ and $(F_e^n)^{-1} \in \mathcal{C}^1(\Omega)$, we apply the non classical Green formula,

$$\begin{aligned}\int_\Omega Div(\mathcal{A}\nabla P^n)(\chi^n(y))\phi(y)\,dy &= \int_\Gamma (F_e^n)^{-T}(y)\vec{n}(y)\cdot(\mathcal{A}\nabla P^n)(\chi^n(y))\phi(y)\,dA \\ &- \int_\Omega (F_e^n)^{-1}(y)(\mathcal{A}\nabla P^n)(\chi^n(y))\cdot\nabla\phi(y)\,dy \\ &- \int_\Omega Div((F_e^n)^{-T}(y))\cdot(\mathcal{A}\nabla P^n)(\chi^n(y))\phi(y)\,dy,\end{aligned}$$

where $F_e^n = \nabla\chi^n$, $Div((F_e^n)^{-T}(y)) = 0$, $\vec{n}$ is a vector normal to the boundary pointing outward and dA denotes the integration measure on the boundary Γ.

Variational formulation

Find $P^{n+1,k+1} \in H^1(\Omega)$ *such that,*

$$\int_\Omega P^{n+1,k+1}(y)\phi(y)\,dy + \frac{\Delta\tau}{2}\int_\Omega (\mathcal{A}\nabla P^{n+1,k+1})(y)\cdot\nabla\phi(y)\,dy$$
$$+ \frac{\Delta\tau}{2}\int_\Omega I\!P^{n+1,k+1}(y)\phi(y)\,dy = \int_\Omega P^n(\chi^n(y))\phi(y)\,dy$$
$$- \frac{\Delta\tau}{2}\int_\Omega (F_e^n)^{-1}(y)(\mathcal{A}\nabla P^n)(\chi^n(y))\cdot\nabla\phi(y)\,dy - \frac{\Delta\tau}{2}\int_\Omega I\!P^n(\chi^n(y))\phi(y)\,dy$$
$$+ \frac{\Delta\tau}{2}\int_\Gamma (F_e^n)^{-T}(y)\vec{n}(y)\cdot(\mathcal{A}\nabla P^n)(\chi^n(y))\phi(y)\,dA$$
$$+ \frac{\Delta\tau}{2}\int_\Omega \hat{h}^{n+1}(y)\phi(y)\,d\vec{(}y) + \frac{\Delta\tau}{2}\int_\Omega \hat{h}^n(\chi^n(y))\phi(y)\,d\vec{(}y)$$
$$+ \frac{\Delta\tau}{2}\int_\Omega \frac{\beta_g\hat{g}}{2}\frac{\partial\theta^{n+1,k}}{\partial y_2}(y)\phi(y)\,dy + \frac{\Delta\tau}{2}\int_\Omega \frac{\beta_g\hat{g}}{2}\frac{\partial\theta^n}{\partial y_2}(\chi^n(y))\phi(y)\,dy.$$

Using that $(F_e^n)^{-1}(y) = \mathcal{I}(y) + \Delta\tau \mathcal{J}^n(\chi^n(y)) + \mathcal{O}(\Delta\tau^2)$, with $\mathcal{J} = \nabla v$:

$$\int_\Omega P^{n+1,k+1}(y)\phi(y)\,dy + \frac{\Delta\tau}{2}\int_\Omega (\mathcal{A}\nabla P^{n+1,k+1})(y)\cdot\nabla\phi(y)\,dy$$
$$+\frac{\Delta\tau}{2}\int_\Omega IP^{n+1,k+1}(y)\phi(y)\,dy = \int_\Omega P^n(\chi^n(y))\phi(y)\,dy$$
$$+\frac{\Delta\tau}{2}\int_\Omega (\mathcal{A}\nabla P^n)(\chi^n(y))\nabla\phi(y)\,dy - \frac{\Delta\tau}{2}\int_\Omega IP^n(\chi^n(y))\phi(y)\,dy$$
$$-\frac{\Delta\tau}{2}\int_\Omega \Delta\tau\mathcal{J}^n(\chi^n(y))(\mathcal{A}\nabla P^n)(\chi^n(y))\nabla\phi(y)\,dy$$
$$+\frac{\Delta\tau}{2}\int_\Gamma (\mathcal{I}(y)+\Delta\tau\mathcal{J}^n(\chi^n(y)))^T\vec{n}(y)\cdot(\mathcal{A}\nabla P^n)(\chi^n(y))\phi(y)\,dA$$
$$+\frac{\Delta\tau}{2}\int_\Omega \hat{h}^{n+1}(y)\phi(y)\,d\vec{(}y) + \frac{\Delta\tau}{2}\int_\Omega \hat{h}^n(\chi^n(y))\phi(y)\,d\vec{(}y)$$
$$+\frac{\Delta\tau}{2}\int_\Omega \frac{\beta_g\hat{g}}{2}\frac{\partial\theta^{n+1,k}}{\partial y_2}(y)\phi(y)\,dy + \frac{\Delta\tau}{2}\int_\Omega \frac{\beta_g\hat{g}}{2}\frac{\partial\theta^n}{\partial y_2}(\chi^n(y))\phi(y)\,dy.$$

Spatial discretization finite elements

We introduce the finite elements space

$$\nu_h = \{\psi_h \in \mathcal{C}^0(\Omega) | \, \psi_{h\mathcal{T}} \in \mathcal{Q}_2, \quad \forall \mathcal{T} \in \tau_h\},$$

where

- $\{\tau_h\}$ is a family of quadrangular meshes of the domain Ω,
- $\mathcal{Q}_2$ is the space of polynomials defined in $\mathcal{T} \in \tau_h$ with degree less or equal than two in each spatial variable,
- $\mathcal{C}^0(\Omega)$ is the space of piecewise continuous functions on Ω.

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Real case example

We have used the **real New Jersey market data** presented for SREC markets:

- M. Coulon, J. Khazaei, W. B. Powell, SMART-SREC: A stochastic model of the New Jersey solar renewable energy certificate market, *Journal of environmental Economics and Management*, 73 (2015), 13-31.

Parameters in the PDE model.

Parameter	T	γ	α_g	β_g	σ_g	r
Value	13	3	2	1.27×10^{-3}	0.1863	0.02

Real case example

Parameters in the seasonality function.

Parameter	a_1	a_2	a_3	a_4
Value	-0.1209	0.0900	0.2151	0.3859

Requeriments and penalty values for each energy year.

Energy year	R_i	Π_i
2011	306000	675
2012	442000	658
2013	596000	641

Real case example

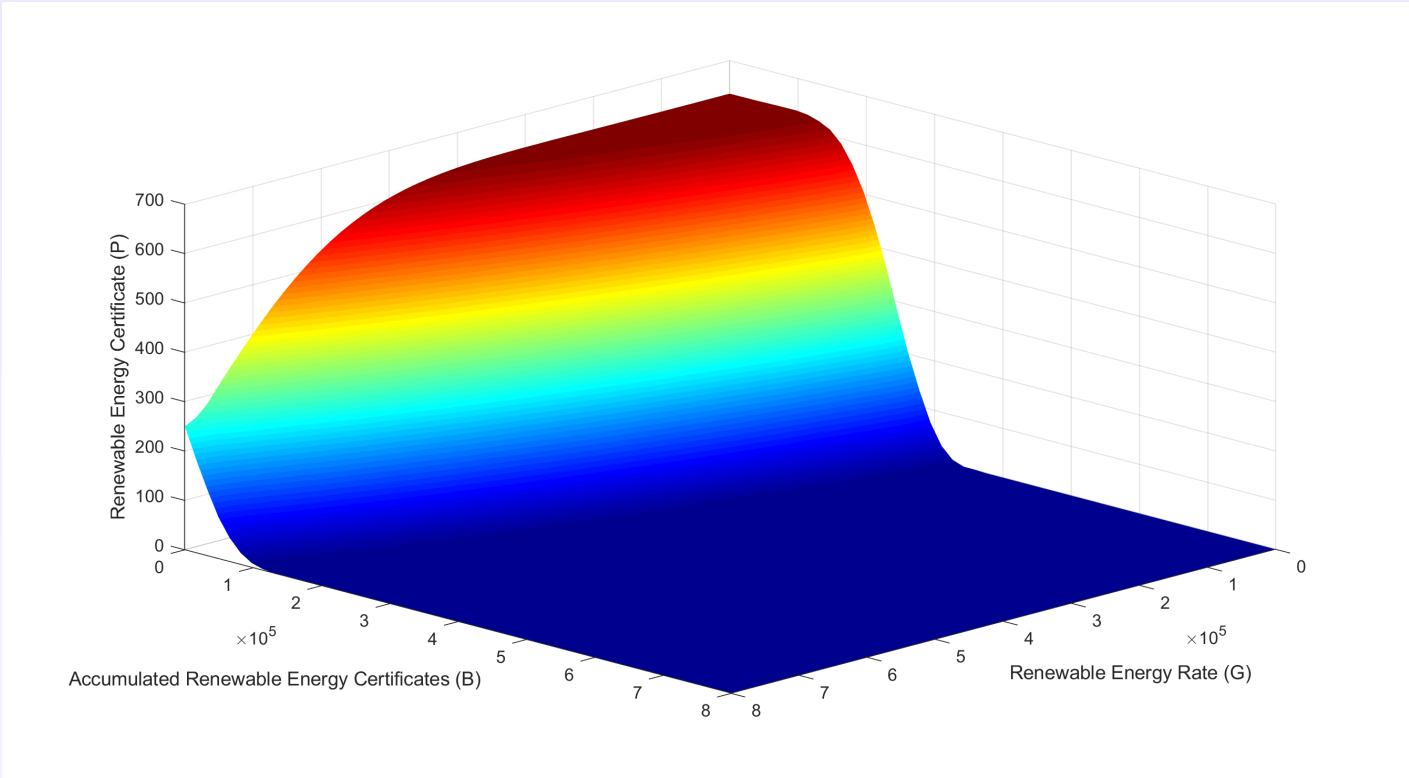


Figure: Renewable energy certificate price at time $t = T - 2/3$ in the real test.

Real case example

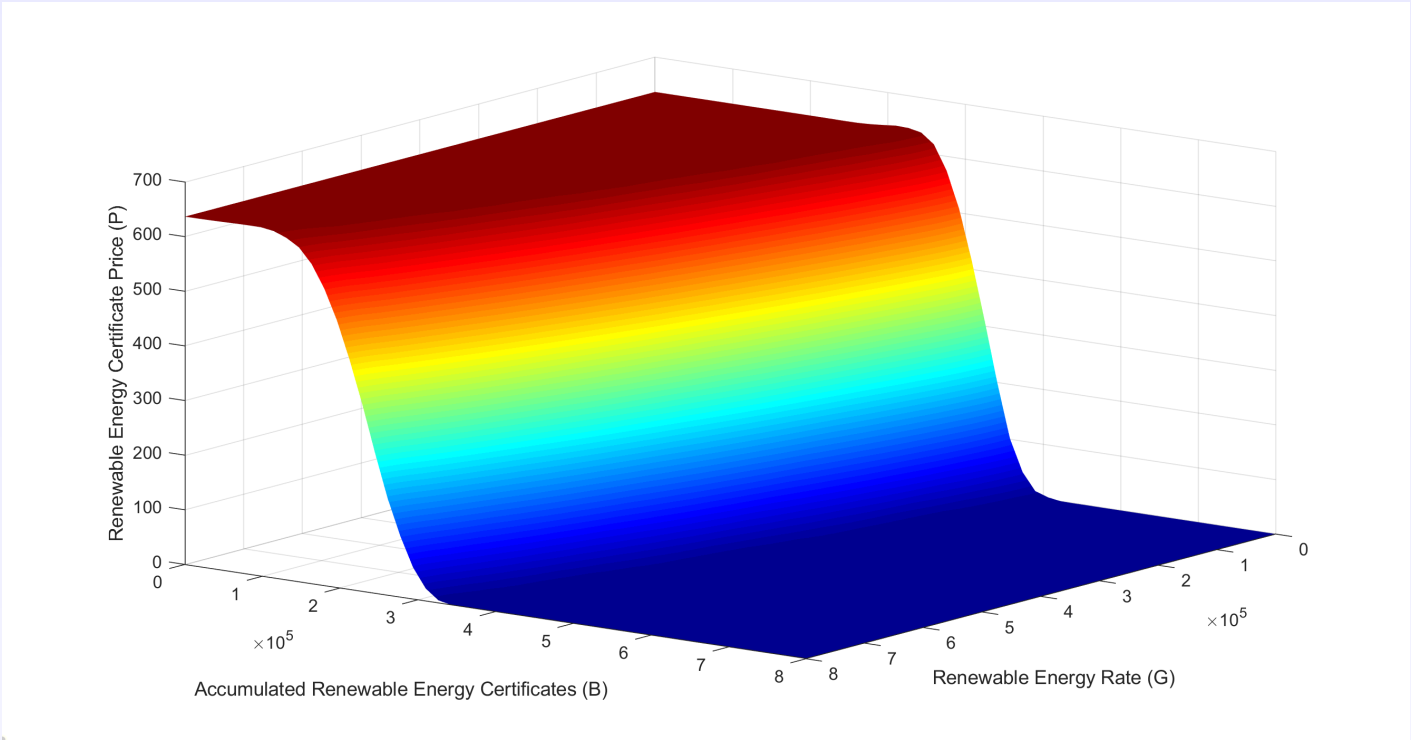


Figure: Renewable energy certificate price at time $t = T - 1/3$ in the real test.

Real case example

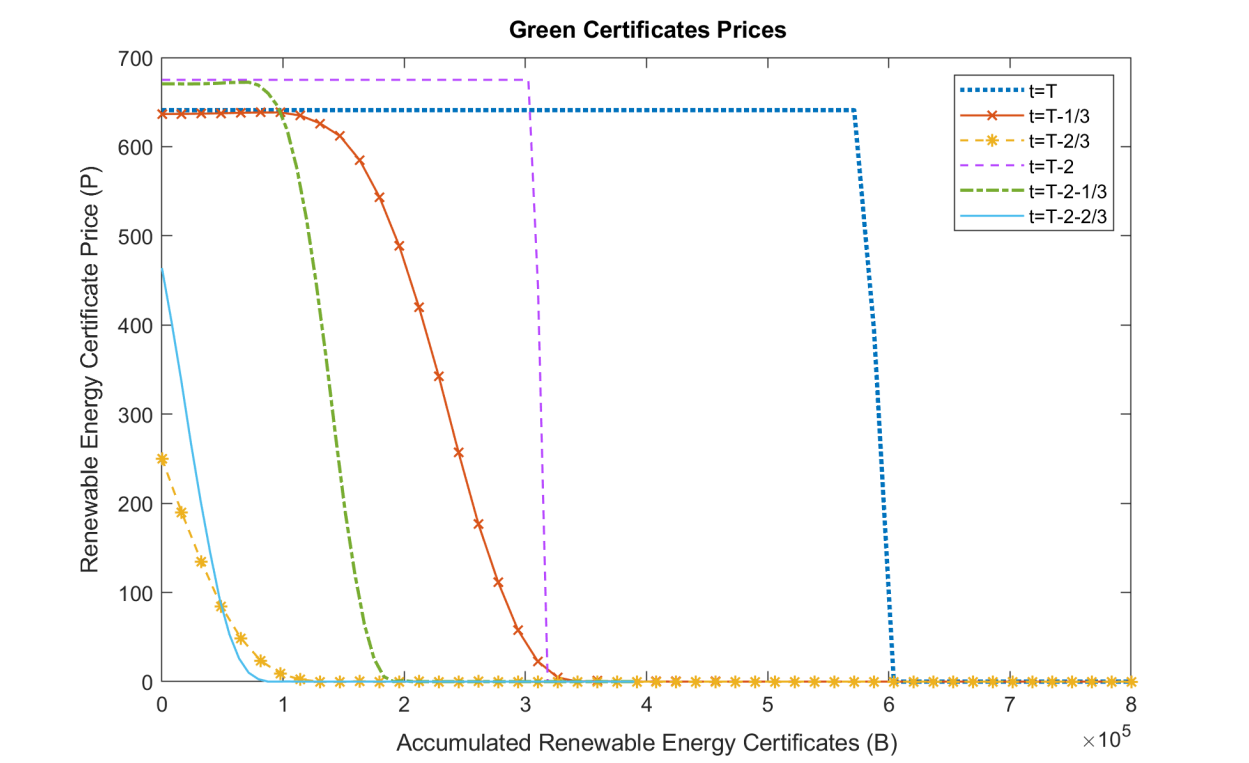


Figure: **Price curves for different times.**

Reference

For further details, see:

M. A. Baamonde-Seoane, M. C. Calvo-Garrido, C. Vázquez, Pricing renewable energy certificates with a Crank-Nicolson Lagrange-Galerkin numerical method, *Journal of Computational and Applied Mathematics*, 422 (2023) 114891.

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Energy derivatives

An *energy derivative* is a contract that depends on an underlying energy-related commodity. Such a contract may be an agreement to trade a commodity at some future date or to exchange cashflows based on energy prices at future dates.

We will focus on European options.

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Underlying stochastic factors

We assume the existence of a continuous, $\mathcal{F}$-adapted process $\boldsymbol{V_t} = \boldsymbol{V}(\boldsymbol{t}, \boldsymbol{B_t}, \boldsymbol{G_t})$, which represents the price of a derivative on the REC, depending on the stochastic factors:

- $\boldsymbol{G_t}$, the renewable generation rate,
- $\boldsymbol{B_t}$, the number of accumulated green certificates.

We assume that:

- the values of these two factors are known at the **initial time** $t = t_0$,
- the time structure of the **REC derivative** is the same as the REC: it starts at $t_0 = T - \gamma$ and the value of the REC derivative is unknown at $t = t_0$.

Generic derivative

The **value of the derivative at maturity** (payoff) is given by:

$$V_T = \zeta(K, P_T),$$

where $\zeta : \mathbb{R}^+ \longrightarrow \mathbb{R}^+$ is a bounded measurable function, $\boldsymbol{P_T}$ is the REC value at maturity and $\boldsymbol{K}$ is the strike price of the derivative.

Analogously to the pricing of the REC, the **price of a European generic derivative** at any time t before expiry is given by the continuous version of the uniformly integrable martingale

$$V_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}\left[\zeta(K, P_T)|\mathcal{F}_t\right],$$

where $\boldsymbol{r}$ is the constant risk free interest rate.

Single period case

The **price of a generic REC derivative V_t** in single period case is given by the solution of the following FBSDE in the interval $[t_0, T]$:

$$\begin{cases} d\widetilde{G}_t = \mu_g(t, \widetilde{G}_t, P_t)\, dt + \sigma_g\, dW_t^0, & \widetilde{G}_{t_0} = g_0, \\ dB_t = G_t\, dt, & B_{t_0} = 0, \\ dP_t = rP_t\, dt + Z_t^0\, dW_t^0, & P_T = \psi(B_T), \\ dV_t = rV_t\, dt + Z_t^1\, dW_t^1, & V_T = \zeta(K, P_T), \end{cases}$$

for some $\mathcal{F}_t$-adapted square integrable process Z_t^1 and Brownian motion W_t^1.

Single period case

We assume a given function $\boldsymbol{P}$, that represents the value of the REC.

Using Itô's formula for a process $V_t = V(t, B_t, \widetilde{G}_t)$, the following linear PDE for the function $V = V(t, B, \widetilde{G})$ is obtained:

$$\mathcal{L}_2[V] = \frac{\partial V}{\partial t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 V}{\partial \widetilde{G}^2} + \mu_g(t, \widetilde{G}, P)\frac{\partial V}{\partial \widetilde{G}} + \exp(\widetilde{G})\frac{\partial V}{\partial B} - rV = 0,$$

which defines the final value problem jointly with the final condition

$$V(T, B, \widetilde{G}) = \zeta(K, P(T, B, \widetilde{G})).$$

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Existence of solution of the PDE

We introduce the new variables $x = (x_1, x_2)$, where

$$x_1 = \exp(\widetilde{G}), \quad x_2 = B.$$

Thus, the previous PDE can be expressed as

$$\frac{\partial V}{\partial t} + x_1 \left(\frac{1}{2}\sigma_g^2 + \mu_g(t, \log(x_1), P) \right) \frac{\partial V}{\partial x_1} + \frac{1}{2}\sigma_g^2 x_1^2 \frac{\partial^2 V}{\partial x_1^2} + x_1 \frac{\partial V}{\partial x_2} - rV = 0,$$

where $P = P(t, x_1, x_2)$.

Associated to PDE, we consider the differential operator

$$\mathcal{L} := \frac{1}{2}\sum_{i,j=1}^{p_0} a_{ij}(t,x)\partial_{x_i x_j} + \sum_{i=1}^{p_0} a_i(t,x)\partial_{x_i} + < Hx, \nabla_x >, \ t \in [0,T),$$

$x \in D \subseteq \mathbb{R}^d$, where $D = (0,+\infty)\times(0,+\infty) \subset \mathbb{R}^2$, $p_0 = 1$, $d = 2$ and H is a (2×2)-matrix with constant real entries:

$$a_{11}(t,x) = \sigma_g^2 x_1^2,$$

$$a_1(t,x) = x_1\left(\frac{1}{2}\sigma_g^2 + \mu_g(t,\log(x_1),P)\right),$$

$$H = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}.$$

$p_0 < d$ implies that no ellipticity condition on $\mathcal{L}$ is satisfied.

- A. Lanconelli, S. Pagliarani, A. Pascucci, Local densities for a class of degenerate diffusions, *Ann. Inst. H. Poincaré Probab. Statist.*, 56 (2), 1440-1464 (2020).

Theorem 4.4.5.

Let X be a local diffusion on $\mathbb{R}^d$ generated by $\mathcal{L}$ on D and let Assumptions 4.4.1. and 4.4.2. be in force. Then:

a) X has a local transition density Γ on D, namely a non-negative measurable function $\Gamma(t, x; T_1, y)$ defined for any $t_0 < t < T_1 < T$ and $x \in \mathbb{R}^2$, $y \in D$, such that

$$p(t, x; T_1, A) = \int_A \Gamma(t, x; T_1, y)\, dy, \quad A \in \mathcal{B}(D).$$

Furthermore, $\Gamma(t, x; T_1, \cdot)$ is continous on D and locally bounded uniformly w.r.t. $x \in \mathbb{R}^d$.

Theorem 4.4.5. (continuation)

b) If $M \geq 2$, then for any $(t, x) \in (t_0, T) \times \mathbb{R}^2$ the function $\Gamma(t, x; \cdot, \cdot) \in C_H^{M,\alpha}((t, T) \times D)$ and solves the forward Kolmogorov equation

$$\mathcal{L}^* u = 0, \qquad \text{on } (t, T) \times D,$$

where $\mathcal{L}^*$ is the formal adjoint of $\mathcal{L}$.

c) If Assumption 4.4.4. is also in force, then for any $(T_1, y) \in (t_0, T) \times D$ the function $\Gamma(\cdot, \cdot; T_1, y) \in C_H^{M+2,\alpha}((t_0, T_1) \times D)$ and solves the backward Kolmogorov equation

$$\mathcal{L} u = 0, \qquad \text{on } (t_0, T_1) \times D.$$

Assumption 4.4.1.

The matrix H is such that the Kolmogorov operator

$$\mathcal{K} = \frac{1}{2}\sum_{j=1}^{p_0} \partial_{x_j}^2 + < Hx, \nabla_x > + \partial_t, \quad x \in \mathbb{R}^d,$$

is hypoelliptic on $\mathbb{R} \times \mathbb{R}^d$. Equivalently, the vector fields $\partial_{x_1}, ..., \partial_{x_{p_0}}$ and $Y := < Hx, \nabla_x > + \partial_t$ satisfy the Hörmander condition

$$\operatorname{rank}(\mathfrak{L}(\partial_{x_1}, ..., \partial_{x_{p_0}}, Y)) = d,$$

at each point of $\mathbb{R} \times \mathbb{R}^d$. Here $\mathfrak{L}(\partial_{x_1}, ..., \partial_{x_{p_0}}, Y)$ denotes the Lie algebra generated by $\partial_{x_1}, ..., \partial_{x_{p_0}}, Y$.

In our case ($p_0 = 1,\ d = 2$), $Y = x_1\partial_{x_2} + \partial_t$ and the Kolmogorov operator is

$$\mathcal{K} = \frac{1}{2}\partial_{x_1^2} + x_1\partial_{x_2} + \partial_t,$$

The operator fulfills the following Hörmander condition

$$[\partial_{x_1}, Y] = \partial_{x_1} Y - Y\partial_{x_1} = \partial_{x_2}.$$

Therefore, we have

$$\operatorname{rank}(\mathfrak{L}(\partial_{x_1}, Y)) = 2,$$

so that the previous **Assumption 4.4.1. is satisfied**.

Assumption 4.4.2.

There exist $M \in \mathbb{N}_0$, $\alpha \in (0,1]$ and $k > 0$ such that:

(i) a_{ij}, $a_i \in C_H^{M,\alpha}((t_0, T) \times D)$ for any $i, j = 1, ..., p_0$, with all the (Lie) derivatives bounded by k;

(ii) the following coercivity condition holds on D:

$$k^{-1}|\xi|^2 \leq \sum_{i,j=1}^{p_0} a_{ij}(t,x)\xi_i\xi_j \leq k|\xi|^2, \quad t \in (t_0, T), x \in D, \xi \in \mathbb{R}^{p_0}.$$

- We assume enough regularity on P so that a_1 satisfies i). Clearly, coefficient a_{11} satisfied the condition.
- In order to satisfy the coercivity condition ii) and also condition i), we consider $D = (a, +\infty) \times (0, +\infty)$, with $a > 0$.

Assumption 4.4.4.

X is a Feller process on D, i.e. for any $T_1 \in (t_0, T)$ and bounded $\varphi \in C(\mathbb{R}^d)$ the function $(t, x) \longmapsto E_{t,x}[\varphi(X_{T_1})]$ is continuous on $(t_0, T_1) \times D$.

- A. Lanconelli, S. Pagliarani, A. Pascucci, Local densities for a class of degenerate diffusions, *Ann. Inst. H. Poincaré Probab. Statist.*, 56 (2), 1440-1464 (2020).

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PDE formulation in a bounded domain

The **formulation of the PDE** in the divergence form is:

Find $V : [0, \gamma] \times \Omega \to \mathbb{R}$ *such that*

$$\begin{aligned}
\frac{\partial V}{\partial \tau} - Div(\mathcal{A}\nabla V) + v_P \cdot \nabla V + IV &= \hat{h} \text{ in } (0,\gamma) \times \Omega\,, \\
V(0,\cdot) &= \zeta(K, P_T) \text{ in } \Omega, \\
\frac{\partial V}{\partial y_1} &= 0 \text{ on } (0,\gamma) \times \Gamma_1^+, \\
\frac{\partial V}{\partial y_2} &= 0 \text{ on } (0,\gamma) \times (\Gamma_2^- \cup \Gamma_2^+),
\end{aligned}$$

where the **bounded spatial domain** is $\Omega = (0,1) \times (0,1)$.

PDE formulation in a bounded domain

Moreover, the **diffusion matrix** $\mathcal{A}$, the **velocity field** v_P, the **linear term** l and the **second member function** $\hat{h}$ have the following expressions:

$$\mathcal{A} = \begin{pmatrix} 0 & 0 \\ 0 & \dfrac{\hat{g}^2\sigma_g^2}{2} \end{pmatrix},$$

$$v_P = \begin{pmatrix} -\hat{b}\exp(y_2\hat{g} - \bar{g}) \\ -\hat{g}\alpha_g\left(f(T-\tau) - (y_2\hat{g} - \bar{g}) + \dfrac{\beta_g}{\alpha_g}P(\tau, y_1, y_2)\right) \end{pmatrix},$$

$$l = r,$$

$$\hat{h} = 0.$$

Characteristics method time discretization

Let us define

$$\frac{DV}{D\tau} = \frac{\partial V}{\partial \tau} + v_P \cdot \nabla V,$$

which represents the material derivative along the characteristic curve through $y = (y_1, y_2) = (\hat{B}, \hat{G})$ at time $\bar{s}$. We denote the characteristics curve associated to the velocity field v_P by $\chi(y, \bar{s}; s)$, which is the solution of the following final value ODE problem:

$$\frac{\partial}{\partial s}\chi(y, \bar{s}; s) = v_P(s, \chi(y, \bar{s}; s)), \quad \chi(y, \bar{s}; \bar{s}) = y.$$

Characteristics method time discretization

- Unlike in the RECs pricing problem, since the velocity field v_P depends on P and the characteristic curves cannot be obtained analytically, we consider the explicit Euler scheme to approximate the characteristics curves:

$$\chi^n(y) \approx y - \Delta t \; v_P^{n+1}(y),$$

where $\chi^n(y) = \chi(y, \tau^{n+1}; \tau^n)$.

Crank-Nicolson characteristics scheme

- Crank-Nicolson characteristics time discretization around $(\chi(y, \tau^{n+1}; \tau^{n+\frac{1}{2}}), \tau^{n+\frac{1}{2}})$, $n = 0, \ldots, N_T - 1$.

$$\frac{V^{n+1}(y) - V^n(\chi^n(y))}{\Delta\tau} - \frac{1}{2} Div(\mathcal{A}\nabla V^{n+1})(y) - \frac{1}{2} Div(\mathcal{A}\nabla V^n)(\chi^n(y)) + \frac{1}{2}(I\, V^{n+1})(y) + \frac{1}{2}(I\, V^n)(\chi^n(y)) = \frac{1}{2}\hat{h}^{n+1}(y) + \frac{1}{2}\hat{h}^n(\chi^n(y)).$$

Using that $\nabla Div(v_P(\chi^n(y))) = 0$ and considering the previous approximation of $(F_e^n)^{-1}(y)$, the variational formulation is

$$\int_\Omega V^{n+1}(y)\phi(y)\,dy + \frac{\Delta\tau}{2}\int_\Omega (\mathcal{A}\nabla V^{n+1})(y)\cdot\nabla\phi(y)\,dy$$
$$+\frac{\Delta\tau}{2}\int_\Omega IV^{n+1}(y)\phi(y)\,dy = \int_\Omega V^n(\chi^n(y))\phi(y)\,dy$$
$$+\frac{\Delta\tau}{2}\int_\Omega (\mathcal{A}\nabla V^n)(\chi^n(y))\nabla\phi(y)\,dy - \frac{\Delta\tau}{2}\int_\Omega IV^n(\chi^n(y))\phi(y)\,dy$$
$$-\frac{\Delta\tau}{2}\int_\Omega \Delta\tau\mathcal{J}^n(\chi^n(y))(\mathcal{A}\nabla V^n)(\chi^n(y))\nabla\phi(y)\,dy$$
$$+\frac{\Delta\tau}{2}\int_\Gamma (\mathcal{I}(y)+\Delta\tau\mathcal{J}^n(\chi^n(y)))^T\vec{n}(y)\cdot(\mathcal{A}\nabla V^n)(\chi^n(y))\phi(y)\,dA$$
$$+\frac{\Delta\tau}{2}\int_\Omega \hat{h}^{n+1}(y)\phi(y)\,d\vec{(}y) + \frac{\Delta\tau}{2}\int_\Omega \hat{h}^n(\chi^n(y))\phi(y)\,d\vec{(}y).$$

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Real case data

- We carry out the valuation of the derivatives with **maturity** $\boldsymbol{T = 13}$.
- We assume that the **life of the derivatives**, γ, is one year ($\boldsymbol{\gamma = 1}$) and also that the REC corresponds to the case of a single compliance period.
- We consider the **strike price** $\boldsymbol{K = \pi_T/2}$, where $\boldsymbol{\pi_T}$ is the **penalty**.
- For the **parameters** $\boldsymbol{\alpha_g}$, $\boldsymbol{\beta_g}$, $\boldsymbol{\sigma_g}$ and $\boldsymbol{r}$ appearing in the PDE, we consider the following:

Parameter	T	γ	α_g	β_g	σ_g	r
Value	13	1	2	1.27×10^{-3}	0.1863	0.02

REC call option

The **price of European vanilla call option on a REC** at time t is given by $V_t^C = V^C(t, B_t, \widetilde{G}_t)$ and its payoff at maturity V_T^C is given by

$$V_T^C = (P_T - K)^+,$$

where $K \geq 0$ is the strike price.

The formulation of the call option pricing problem is

$$\mathcal{L}_2\left[V^C\right] = \frac{\partial V^C}{\partial t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 V^C}{\partial \widetilde{G}} + \alpha_g \left(f(t) + \frac{\beta_g}{\alpha_g} P - \widetilde{G}\right) \frac{\partial V^C}{\partial \widetilde{G}} + \exp(\widetilde{G}) \frac{\partial V^C}{\partial B} - rV^C = 0,$$

jointly with the final condition

$$V^C(T, B, \widetilde{G}) = (P(T, B, \widetilde{G}) - K)^+.$$

REC call option

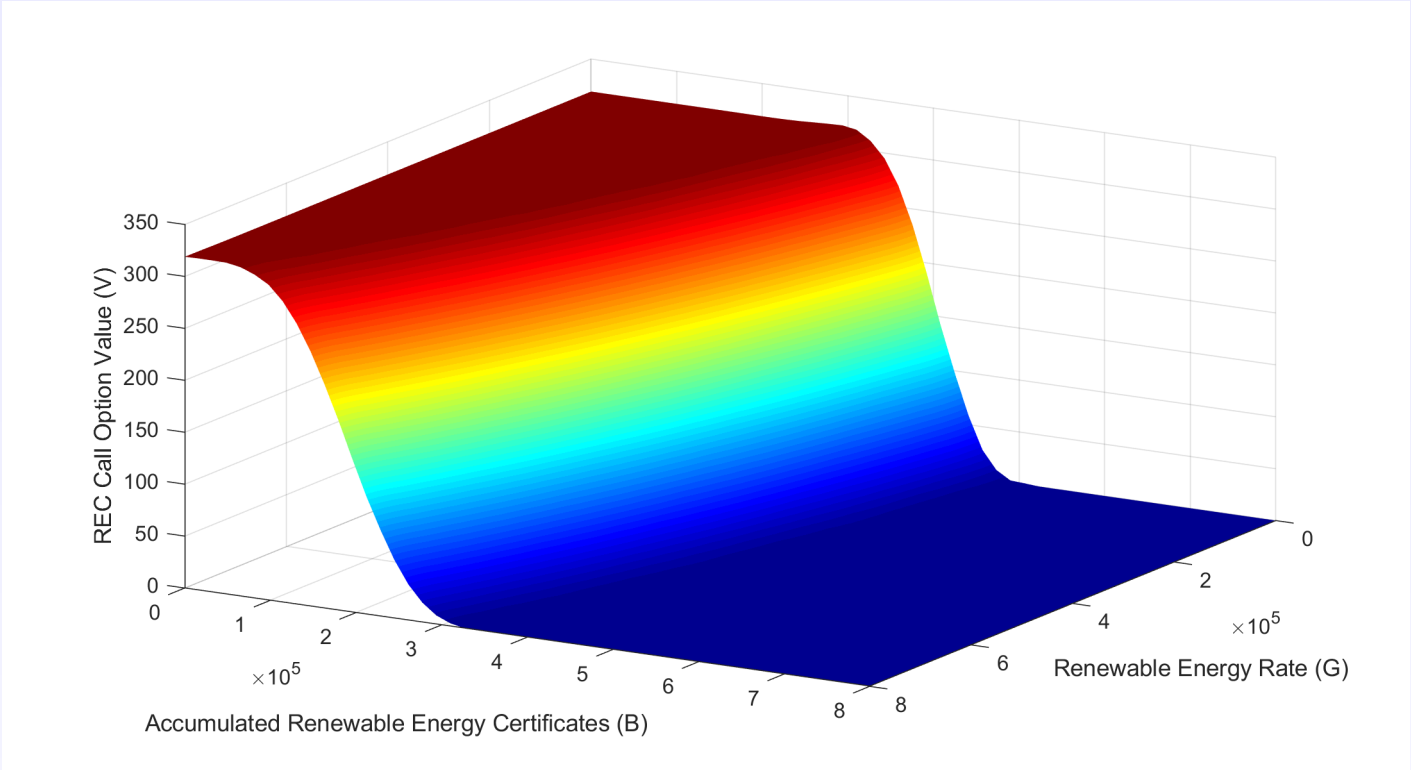


Figure: REC call option price surface at $t = T - 1/3$.

REC call option

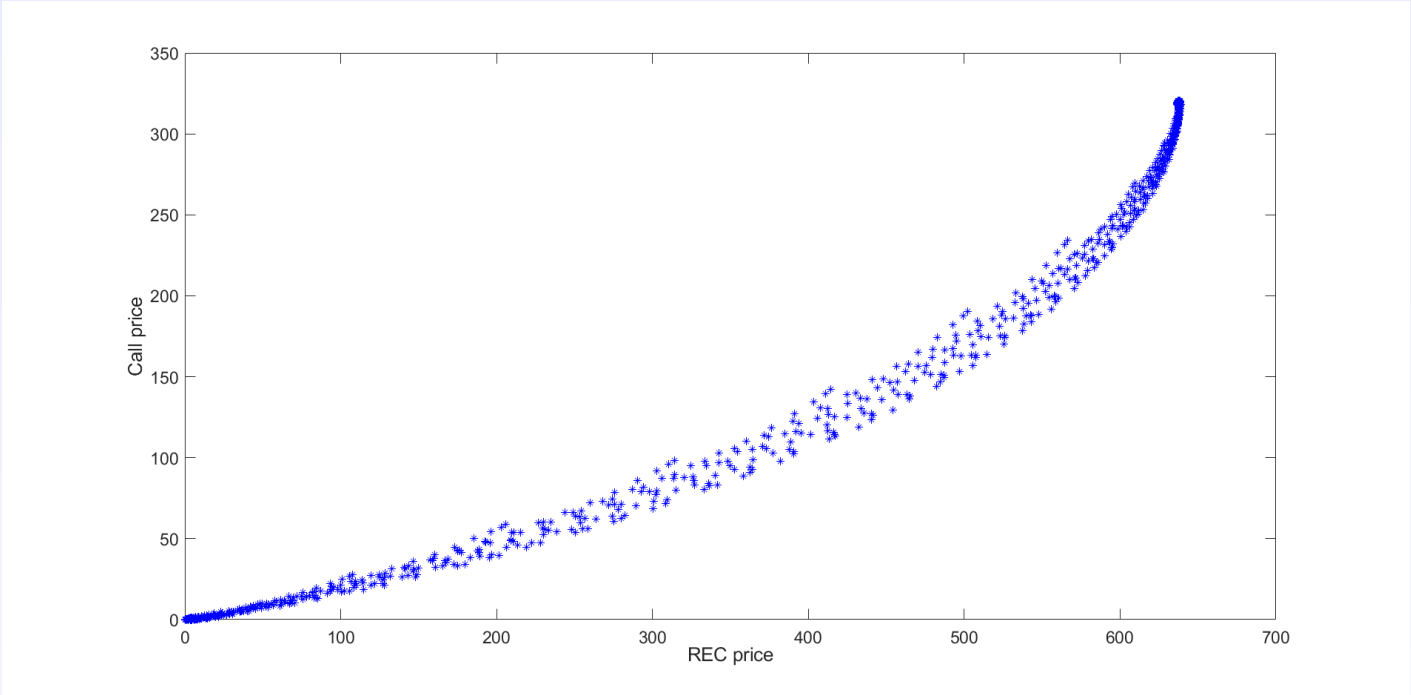


Figure: REC call option price versus REC price at time $t = T - 1/3$.

Reference

For further details, see:

M. A. Baamonde-Seoane, M. C. Calvo-Garrido, C. Vázquez, Models and numerical methods for pricing renewable energy certificate derivatives, *Communications in Nonlinear Science and Numerical Simulation*, 118 (2023) 107066.

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Jumps in the dynamics of renewable generation rate

The renewable generation rate, $\boldsymbol{G_t}$, satisfies $G_t = \exp(\widetilde{G}_t)$, with:

$$d\widetilde{G}_t = \alpha_g \left(f(t) + \frac{\beta_g}{\alpha_g} P_t - \widetilde{G}_t \right) dt + \sigma_g dW_t^0 + J dN_t, \qquad \widetilde{G}_{t_0} = g_0,$$

- α_g is the mean reversion speed of the process,
- $f(t)$ is the seasonality function,
- β_g is the parameter which controls the level of feedback from the price of the certificate,
- σ_g is the volatility,
- W_t^0 is an $\mathcal{F}_t$-adapted $\mathbb{Q}$-Brownian motion,
- J is the random jump size with PDF ν and expectation κ,
- N_t is a Poisson process with intensity λ.

Single period case with jumps in generation rate

Using Itô's formula for jump-diffusion processes $P_t = P(t, B_t, \widetilde{G}_t)$, the following non-linear PIDE for the function $P = P(t, B, \widetilde{G})$ can be obtained:

$$\begin{aligned}\mathcal{L}_1[P] \quad = \quad & \frac{\partial P}{\partial t} + \frac{1}{2}\sigma_g^2 \frac{\partial^2 P}{\partial \widetilde{G}^2} + \left(\mu_g(t, \widetilde{G}, P) - \lambda\kappa\right) \frac{\partial P}{\partial \widetilde{G}} + \exp(\widetilde{G}) \frac{\partial P}{\partial B} \\ & -(r+\lambda)P + \lambda \int_{-\infty}^{+\infty} P(t, B_t, G_{t-} \exp(z))\nu(z)dz = 0,\end{aligned}$$

which defines the final value problem jointly with the condition

$$P(T, B, \widetilde{G}) = \pi_T \mathbb{1}_{[0, R_T)}(B),$$

where

$$\mu_g(t, \widetilde{G}, P) = \alpha_g \left(f(t) + \frac{\beta_g}{\alpha_g} P - \widetilde{G} \right).$$

Single period case with jumps in generation rate

We introduce the new variables $x = (x_1, x_2)$, where

$$x_1 = G = \exp(\widetilde{G}), \quad x_2 = B.$$

Thus, the previous non-linear PIDE can be expressed as

$$\frac{\partial P}{\partial t} + \left(\alpha_g \left(f(t) + \frac{\beta_g}{\alpha_g} P - \log x_1 \right) + \frac{1}{2}\sigma_g^2 - \lambda\kappa \right) x_1 \frac{\partial P}{\partial x_1} + x_1 \frac{\partial P}{\partial x_2}$$
$$+ \frac{1}{2}\sigma_g^2 x_1^2 \frac{\partial^2 P}{\partial x_1^2} - (r+\lambda)V + \lambda \int_{-\infty}^{+\infty} P(t, \log x_1 \exp(z), x_2)\, \nu(z) dz = 0,$$

where $P = P(t, x_1, x_2)$.

Single period case with jumps in generation rate

So, by using the same duality method as in the case without jumps, we pose:

$$\mathcal{L}_J[P] = -\frac{\beta_g}{2}\frac{\partial\theta}{\partial x_1}, \quad \theta = m_\lambda^\omega(P + \lambda\theta),$$

where we have introduced the linear PIDE operator

$$\mathcal{L}_J[P] = \frac{\partial P}{\partial t} + \left(\alpha_g(f(t) - \log x_1) + \frac{\beta_g\omega}{2} + \frac{1}{2}\sigma_g^2 - \lambda\kappa\right) x_1\frac{\partial P}{\partial x_1} + x_1\frac{\partial P}{\partial x_2}$$
$$+\frac{1}{2}\sigma_g^2 x_1^2\frac{\partial^2 P}{\partial x_1^2} - (r+\lambda)V + \lambda\int_{-\infty}^{+\infty} P(t, \log x_1 \exp(z), x_2)\,\nu(z)dz$$

So, we can use the same numerical methods as in the case without jumps and treat numerically the additional integral term

Single period case with jumps in generation rate

Summary of numerical methods:

- Time discretization with Crank Nicholson characteristics for linear PIDE.
- Fixed point iteration between linear PIDE for P and multiplier θ update.
- Integral term: explicit treatment (Adams-Bashford) plus appropriate truncation of the unbounded spacial domain.
- Spatial discretization with finite elements.

Single period case with jumps in generation rate

Numerical results for $T - 1/3$:

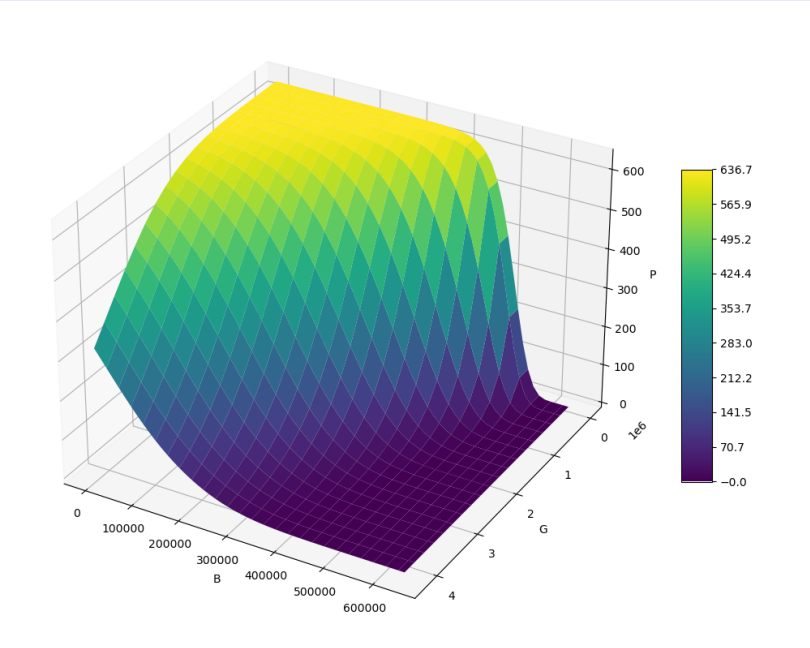


Figure: REC prize for $\lambda = 5$

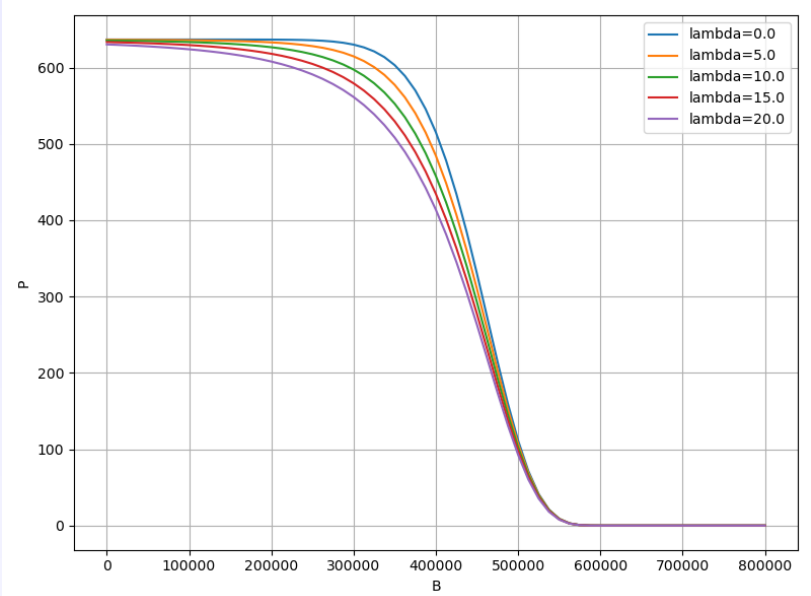


Figure: REC prize for $G = 800000$ and different λ

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Conclusions

- We have proposed **new PDE formulations** for the **pricing of RECs** and some **European derivatives of RECs**.
- We propose a **Bermúdez-Moreno duality method** to treat the **non-linearity in the PDE** for the pricing of RECs.
- We have used a **Crank-Nicolson Lagrange-Galerkin method** for the numerical solution of the resulting linear problem.
- In **a real case**, we have obtained the **price of a REC** and **the price of some REC derivatives**.
- **New PIDE formulations** for the pricing of RECs with jump-diffusion models for generation rate dynamics

Thank you for your attention!