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On the stabilisation of the incompressible Navier Stokes equations in a 2-d channel with a normal control on the boundary, old and new

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In this talk, I will discuss the stabilization of incompressible Navier-Stokes equations in a 2d channel around a fluid at rest when the control acts only on the normal component of the proper subset of the upper boundary. In this case, the linearized equations are not controllable nor stabilizable at an exponential rate higher than π^2/L^2, when the channel is of width L and of length 2π and the viscosity parameter is set to 1. Our main result allows to go above this threshold and reach any exponential decay rate by using the non-linear term to control the directions which are not controllable for the linearized equations. Our approach therefore relies on writing the controlled trajectory as an expansion of order two taking the form $\varepsilon a + \varepsilon^2 \beta$ for $\varepsilon > 0$ small enough. In particular, we can prove that, for the linearized system, only the 0-mode cannot be controlled and that the other modes are null-controllable when the control acts on the whole upper boundary, and (at least) approximately controllable when the control acts on a localized part of the upper boundary. We thus can develop a non-linear strategy to control a finite number of components of the zero modes through the convective terms of the other modes. This strategy was developed in a prior work with Shirshendu Chowdhury when the non-zero modes are null-controllable. I will explain that it can also be adapted when the non-zero modes are only (known to be) approximately controllable, making the strategy more robust.

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