Stability of nonlinear Dirac solitons under the action of external potentials

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The nonlinear Dirac equation in 1+1-dimensions supports localized solitons. Theoretically, these traveling waves propagate with constant velocity, energy, momentum, and charge. However, the soliton profiles can be distorted, and eventually destroyed, due to intrinsic or numerical instabilities. The constants of motion and the initial profiles can also be modified by external potentials, which may give rise to instabilities.
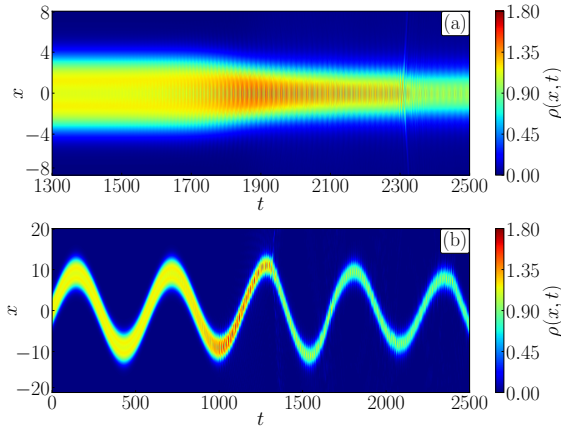

Figure 1: Evolution of the density of the charge from simulations of the ABS model (a) without and (b) with harmonic potential.

In this work [1], we study the instabilities observed in numerical simulations of the Gross-Neveu equation [2] under linear and harmonic potentials. We perform an algorithm [3] based on the method of characteristics to numerically obtain the two soliton spinor components. All studied solitons are numerically stable, except the low-frequency solitons oscillating in the harmonic potential over long periods of time. These instabilities are identified by the non-conservation of both energy and charge, and can be removed by imposing absorbing boundary conditions. We find that the dynamics of the soliton is in perfect agreement with the prediction obtained using an Ansatz with only two collective coordinates. By applying the same methodology, we also demonstrate the spurious character of the reported instabilities in the Alexeeva–Barashenkov–Saxena (ABS) model [4] under external potentials.

Acknowledgements

References

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