Decay rates to solutions of some dissipative systems in Sobolev critical spaces

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Understanding how quickly solutions decay as time approaches infinity is essential for capturing how systems stabilise, how rapidly perturbations vanish, and whether the solutions efficiently reach equilibrium. This understanding provides a link between transient dynamics and the system's long-term behaviour.

In this talk, I will explore the decay rates of solutions in critical Sobolev spaces for a range of dissipative systems. I will present recent results concerning the Navier-Stokes equations, the Navier-Stokes–Coriolis system, the energy-critical nonlinear heat equation, and the Hardy–Sobolev parabolic equation.

The decay estimates are expressed in terms of the decay character of the initial data, yielding algebraic decay rates and showing in detail the roles played by the linear and nonlinear parts. The proof is carried on purely in the critical space. This is the first instance in which such a method is used for obtaining decay bounds in a critical space for nonlinear equations.

In collaboration with M. Ikeda (Japan), L. Kosloff (Brazil), and C.J. Niche (Brazil).

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