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On Keller–Segel models with positive total flux: analytic and modeling perspectives

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Since the introduction of the seminal Keller–Segel models, which provide a mathematical framework for describing chemotaxis—the movement of cells or organisms in response to chemical gradients—there has been an explosion of research on various extensions and modifications of these models. These studies have contributed significantly to our understanding of pattern formation, aggregation, and the dynamics of cell populations under the influence of chemical signals. A unifying feature across the majority of this body of work is the imposition of zero-flux (Neumann-type) boundary conditions on the cell density equation, effectively modeling impenetrable domain boundaries that prevent cell escape or entry.

In this talk, we depart from this standard assumption and explore chemotaxis models under the novel premise that the total cell flux across the boundary is strictly positive, thereby modeling domains that are penetrable. Such a setting introduces new analytical challenges and potential biological interpretations, especially in contexts where cell inflow or outflow plays a significant role. We will present some preliminary results in this direction and discuss open questions and considerations that arise in studying these models.

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References

[1] K. Baghaei, S. Frassu, Y. Tanaka, G. Viglialoro, *To what extent does the consideration of positive total flux influence the dynamics of Keller–Segel-type models?* Submitted.

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