WORKSHOP ON PDES AND CONTROL 2025 (PKM-60)
September 3-5, 2025, Sevilla, Spain

An optimal control problem related to a 3D chemo-repulsion model with nonlinear production

F. Guillén-González [*], M.A. Rodríguez-Bellido [†], E. Mallea-Zepeda[‡] and E.J. Villamizar-Roa [§]

Chemotaxis corresponds to the directional movement of cells or living organisms influenced by the concentration of the chemical signal substance. This motion can be towards a higher (attractive) or lower (repulsive) concentration of the chemical stimuli. In this paper we are interested in the repulsive chemotaxis scenario, in which the presence of living organisms produce chemical substance, including or not a logistic growth of organisms. Explicitly, considering $Q := (0,T)\times\Omega$, with $\Omega\subset\mathbb{R}^3$ being a bounded domain and $(0,T)$, $T>0$, a time interval, we consider the following system

$$(1)\qquad \begin{cases} \partial_t u - \Delta u &= \nabla\cdot(u\nabla v) + ru - \mu u^p,\\ \partial_t v - \Delta v + v &= u^p + f v\, 1_{\Omega_c}, \end{cases}$$

where the unknowns are $u(t,x)\geq 0$ and $v(t,x)\geq 0$ denoting the cell density of some chemotactically active species and the chemical concentration. The nonlinear term $\nabla\cdot(u\nabla v)$, on the right-hand-side (1), describes the repulsion mechanism. In addition, $1\leq p<+\infty$, $r,\mu\geq 0$, and $f=f(t,x)$ is the control function acting on a subdomain $Q_c=(0,T)\times\Omega_c\subset Q=(0,T)\times\Omega$. This system is endowed with initial and non-flux boundary conditions. We prove the existence of global weak solutions when $f\in L^{5/2}(Q_c)$, analyzing the role of the diffusion and the logistic terms to get energy estimates.

Definition 1 (Weak solutions) Let $f\in L^{5/2}(Q_c) := L^{5/2}(L^{5/2}(\Omega_c))$, $(u_0,v_0)\in L^p(\Omega)\times H^1(\Omega)$, with $u_0\geq 0$ and $v_0\geq 0$ a.e. in Ω. A pair (u,v) is called weak solution in $[0,T]$ of system (1) with initial data (u_0,v_0) if $u\geq 0$ and $v\geq 0$ a.e. in Q, $u\in L^\infty(L^p)\cap L^{5p/3}(Q)$, $\quad v\in L^\infty(H^1)\cap L^2(H^2)$, and $u\in L^{2p-1}$ for $p>3$ in the logistic case ($\mu>0$), and $\nabla u\in L^{\gamma(p)}(Q)$ with

$$\gamma(p)=\begin{cases} 5p/(3+p) & \text{when } 1<p\leq 2,\\ 25p/(18+5p) & \text{when } 2<p<12/5,\\ 2 & \text{when } p\geq 12/5,\end{cases}$$

satisfying the u-equation $(1)_1$ in a variational sense, and the v-equation $(1)_1$ holds a.e $(t,x)\in Q$.

[*]Universidad de Sevilla-Spain. Email: guillen@us.es
[†]Universidad de Sevilla-Spain. Email: angeles@us.es
[‡]Universidad de Tarapacá-Chile. Email: emallea@uta.cl
[§]Universidad Industrial de Santander-Colombia. Email: jvillami@uis.edu.co

Theorem 2 (Existence of weak solutions) *Assume that $f\in L^{5/2}(Q_c)$, $(u_0,v_0)\in L^p(\Omega)\times H^1(\Omega)$, with $u_0\geq 0$ and $v_0\geq$ a.e. in Ω. If $1<p\leq 5/3$, then there exists a weak solution of system (1) with $\mu=r=0$, and if $p>1$, then there exists a weak solution of system (1).*

Knowing the existence of global weak solutions, we establish a regularity criterion through which weak solutions of systems become strong solutions. These strong solutions will give the adequate framework to study the following optimal control problem:

$$(2)\qquad \begin{cases} \min J(u,v,f),\\ \text{subject to } (u,v,f)\in\mathcal{S}_{ad}, \end{cases}$$

where $J: L^{5p/2}(Q)\times L^2(Q)\times L^{5/2}(L^{5/2+}(\Omega_c))\to\mathbb{R}$ is the cost functional defined by

$$J=\frac{\gamma_u}{5p/2}\int_0^T\|u-u_d\|_{L^{5p/2}}^{5p/2}dt+\frac{\gamma_v}{2}\int_0^T\|v-v_d\|^2dt+\frac{\gamma_f}{5/2}\int_0^T\|f\|_{L^{5/2+}(\Omega_c)}^{5/2}dt,$$

for some desired states (u_d,v_d). We prove the existence of global optimal solutions and derive first-order necessary optimality conditions for local optimal solutions. All the results presented here were obtained in [1].

Acknowledgements

F. Guillen-Gonzalez and M.A. Rodríguez-Bellido have been partially supported by Grant I+D+I PID2023-149182NB-I00 funded by MICIU/AEI/10.13039/501100011033 and, ERDF/EU. E. Mallea-Zepeda has been supported by ANID-Chile, Project 11200208. E.J. Villamizar-Roa has been supported by VIE-UIS, Project 4210.

References

[1] F. Guillén-González, E. Mallea-Zepeda, M.A. Rodríguez-Bellido, E.J. Villamizar-Roa, *Optimal bilinear control restricted to the three-dimensional chemo-repulsion model with nonlinear production*. Preprint, 2025.