BOUNDARY NULL CONTROLLABILITY OF A CLASS OF 2-D DEGENERATE PARABOLIC PDES

JOINT WORK WITH VÍCTOR HERNÁNDEZ-SANTAMARÍA AND SUBRATA MAJUMDAR

Luz de Teresa

Workshop on PDE's and control (PKM60), Sevilla, septiembre de 2025

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MANOLO

CIRM-1997

	R. Rolland (Maitre Conf., Univ. Aix-Marseille II)
5-9 mai	**Colloque franco-allemand d'algèbre commutative et géométrie algébrique**
Organisateur(s)	M. Chardin (Ch. Rech., Ecole Polytechnique), H. Flenner (Prof., Univ. Goettingen), C. Huneke (Prof., Purdue Univ.) J.-P. Joanolou (Prof., ULP, Strasbourg),
12-16 mai	**Aspects de la géométrie algébrique**
Organisateur(s)	O. Debarre (Prof., ULP, Strasbourg), C. Voisin (Dir. Rech., Univ. Paris 7)
20-23 mai	**Scattering**
Organisateur(s)	J.-M. Combes (Prof., Univ. Toulon)
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Organisateur(s)	B. Baijanov et E. Baisalov (Prof., Univ. Nat. duQazaqstan) Q. Jetpisov (Prof., Univ. Qaragandy), B. Poizat (Prof., Univ. Lyon 1),
2-6 juin	**Méthodes géométriques en physique**
Organisateur(s)	M. Slupinski (Maitre Conf., ULP, Strasbourg) T. Wurzbacher (Maitre Conf., ULP, Strasbourg)
9-13 juin	**Systèmes dynamiques en plusieurs variables complexes**
Organisateur(s)	N. Sibony (Prof., Univ. Paris-Sud)
16-20 juin	**Equations aux dérivées partielles et contrôle**
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Figure: We meet in 1997

de Teresa, Luz

AMS

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MR Author ID	609602
Earliest Indexed Publication	1986
Total Publications	61
Total Related Publications	0
Total Reviews	44
Total Citations	1,067 in 485 publications
Unique Citing Authors	465

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Collaboration Distance

Mathematics Genealogy Project

Published under 5 names

Classifications (8)

Classification	Publications	Citations
93 - Systems theory; control	47	944
35 - Partial differential equations	4	87
91 - Game theory, economics, finance, and other social and behavioral sciences	3	28
49 - Calculus of variations and optimal control; optimization	2	6

Coauthors (40)

Name	Collaborations
González-Burgos, Manuel	12
Benabdallah, Assia	8
Ammar Khodja, Farid	6
Fernández-Cara, Enrique	6
Hernández-Santamaría, Víctor	6

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Figure: 2003

Figure: 2014

Figure: 2014


Controllability of nonlinear parabolic problems in unbounded domains
Manuel González-Burgos,
UNIVERSIDAD DE SEVILLA
60 Years Young: A Conference in Control and PDEs in Honor of
Luz de Teresa
Cuernavaca, June 2025
M. González-Burgos
Controllability unbounded domains

22 years later, same place.

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MR Collaboration Distance = 2

de Teresa, Luz	coauthored with	Fernández-Cara, Enrique	**MR**3395760
Fernández-Cara, Enrique	coauthored with	Guillén-González, Francisco	**MR**1633055

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MAIN PROBLEM

Controllability of a 2-d degenerate parabolic equation

Let us consider $\Omega = (0,1) \times (0,1)$. We study the degenerate parabolic equation in the square.

$$\begin{cases} \partial_t u = \operatorname{div}(D\nabla u) & \text{in } (0,T) \times \Omega, \\ u(t) = 1_\gamma q(t), & \text{in } (0,T) \times \partial\Omega, \\ u(0) = u_0, & \text{in } \Omega. \end{cases} \tag{1}$$

The matrix function $D : \overline{\Omega} \mapsto M_{2\times 2}(\mathbb{R})$ is given by

$$D(x,y) = \begin{pmatrix} x^{\alpha_1} & 0 \\ 0 & y^{\alpha_2} \end{pmatrix},$$

where $\alpha = (\alpha_1, \alpha_2) \in [0,1) \times [0,1)$, and u_0 is the initial data that lies in a functional space $H_\alpha^{-1}(\Omega)$

Null controllability

We say that (1) is null controllable if it exists a control $q \in L^2$ such that

$$u(T) = 0$$

Problems

- Degeneracy and dimension

Problems

- Degeneracy and dimension
- Control on the boundary where the degeneracy takes place

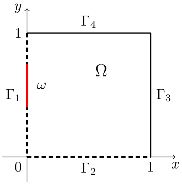

Figure: Control region

Problems

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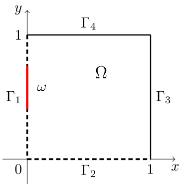

Figure: Control region

- Other boundary conditions depending on the degenerate parameters $\alpha_1, \alpha_2 \subset [0,1) \cup (1,2]$. weak /strong degeneracy.

1-d controllability results

We recall some 1-d results when the degeneracy is of the type $(a(x)u_x)_x$, $a(0) = 0$

- Cannarsa, P.; Martinez, P.; Vancostenoble, J. (2005). Distributed control with support in $\omega \subset (0, 1)$, $a(x) = x^\alpha, \alpha \in [0, 1/2] \cup [5/4, 2)$.
- F. Alabau-Boussouira, P. Cannarsa and G. Fragnelli (2006) Carleman inequalities for general $a(x)$, Again distributed control.
- P. Cannarsa, P. Martinez, J. Vancostenoble (weakly 2017) (strongly 2020). Boundary control (on the degeneracy point).
- Recently, 2024 L. Galo-Mendoza and M. López-García, extended the previous results with the control acting on both ends.

Multi-d controllability results

Both: Distributed control

- P. Cannarsa, P. Martinez, and J. Vancostenoble. Global Carleman estimates for degenerate parabolic operators with applications. Mem. Amer. Math. Soc., 239(1133):ix+209, 2016.
- F. D. Araruna, B. S. V. Araujo, and E. Fernández-Cara. Carleman estimates for some two-dimensional degenerate parabolic PDEs and applications. SIAM J. Control Optim., 57(6):3985–4010, 2019.

MAIN IDEAS

Strategy

We use the strategy introduced in A.Benabdallah, F. Boyer, M. González-Burgos, and G. Olive.[2014]

- We use the moment method to establish a boundary null controllability result of the one-*d* degenerate parabolic system

$$\begin{cases} \partial_t w = \partial_x (x^{\alpha_1} \partial_x w) & \text{in } (0,T)\times(0,1), \\ w(t,0) = h(t) & \text{if } 0 \le \alpha_1 < 1 \\ w(t,1) = 0 & \text{in } (0,T) \\ w(0,x) = w_0(x) & \text{in } (0,1), \end{cases} \tag{2}$$

  in the space $H^{-1}_{\alpha_1}(0,1)$ with a control cost $C_T = Ce^{C/T}$,
- Prove a partial observability result for the adjoint.
- Use Lebeau-Robbiano strategy.

Eigenvalues and eigenvectors

Let $(\lambda_{\alpha_1,k}, \phi_{\alpha_1,k})_{k\in\mathbb{N}}$ be the eigenelement of the following eigenvalue problem

$$\begin{cases} -(y^{\alpha_2}\phi')'(y) = \lambda\phi(y) \qquad y \in (0,1), \\ \begin{cases} \phi(0) = 0 & \text{if } 0 \le \alpha_1 < 1 \\ (y^{\alpha_1}\phi')(0) = 0 & \text{if } 1 \le \alpha_1 < 2 \end{cases}, \\ \phi(1) = 0. \end{cases} \tag{3}$$

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Theorem (Theorem 1.1, Buffe et al. 2024)

Let (λ_j, Φ_j) *be the solution of the eigenvalue problem* (3). *Let* ω *be an open and nonempty subset of* $(0,1)$. *There exists a constant* $C > 0$ *such that*

$$\sum_{\lambda_j \leq \mu} |a_j|^2 \leq C e^{C_1 \frac{1}{(2-\alpha_2)^2} \sqrt{\mu}} \int_\omega \left| \sum_{\lambda_j \leq \mu} a_j \Phi_j \right|^2 \mathrm{d}y \tag{4}$$

for any $\alpha_1 \in [0,2)$, $\{a_j\} \in \mathbb{R}$, *and any* $\mu > 0$.

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Functional setting

Spaces

Denote the space $H^1_\alpha(\Omega) = \{u \in L^2(\Omega) \mid \nabla u \cdot D\nabla u \in L^1(\Omega)\}$, endowed with the norm

$$\begin{aligned} \|u\|^2_{H^1_\alpha(\Omega)} &= \int_\Omega |u|^2 + \int_\Omega |D^{1/2}\nabla u|^2 \\ &= \int_\Omega |u|^2 + \int_\Omega \left(x^{\alpha_1}|\partial_x u|^2 + y^{\alpha_2}|\partial_y u|^2 \right). \end{aligned}$$

We also denote

$$H^2_\alpha(\Omega) := \left\{ u \in H^1_\alpha(\Omega) \mid \operatorname{div}(D\nabla u) \in L^2(\Omega) \right\}. \tag{5}$$

Next we define the space $H^1_{\alpha,0}(\Omega) := \overline{\mathcal{D}_\alpha}^{H^1_\alpha(\Omega)}$, where the space $\mathcal{D}_\alpha$ depends on $\alpha = (\alpha_1, \alpha_2)$

$\mathcal{D}_\alpha := \{v \in C^\infty(\overline{\Omega}) : \operatorname{supp}(v) \subset\subset \Omega\}, \alpha_1, \alpha_2 \in [0, 1)$,

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Hardy inequality

Thanks to Lemmas 4 and 13, Araruna et al. 2019, we can state the following Hardy-Poincaré inequality.

Lemma

Assume that $\alpha_i \in [0,2] \setminus \{1\}$ *for* $i = 1,2$. *Then, there exists a constant* $C = C(\alpha_1, \alpha_2)$ *such that*

$$\int_\Omega x_i^{\alpha_i - 2} |u|^2 \,\mathrm{d}x_1 \,\mathrm{d}x_2 \leq C \int_\Omega x_i^{\alpha_i} \left| \frac{\partial u}{\partial x_i} \right|^2 \mathrm{d}x_1 \,\mathrm{d}x_2, \quad \forall\, u \in H^1_{\alpha,0}(\Omega). \tag{6}$$

One-*d* control

One dimensional control

$$\begin{cases} \partial_t w = \partial_x(x^{\alpha_1}\partial_x w) & \text{in } (0,T)\times(0,1), \\ w(t,0) = h(t) & \text{if } 0 \le \alpha_1 < 1, \text{ in } (0,T), \\ w(t,1) = 0 & \text{in } (0,T) \\ w(0,x) = w_0(x) & \text{in } (0,1), \end{cases} \tag{7}$$

Theorem

Let $T > 0$. Then, for every $w_0 \in H^{-1}_{\alpha_1}(0,1)$, there exists a control $h \in L^2(0,T)$ such that the system (7) satisfies $w(T) = 0$. The control satisfies the following estimate

$$\|h\|_{L^2(0,T)} \le Ce^{\frac{C}{T}} \|w_0\|_{H^{-1}_{\alpha_1}(0,1)}, \tag{8}$$

for some positive constant C which is independent of T.

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Family of exponentials

Theorem (Theorem 1.5, Benabdallah et al. (2014))

Let $\Lambda = \{\Lambda_k\}_{k\geq 1} \subset \mathbb{C}$ *be a complex sequence satisfying the following properties:*

- $\Lambda_k \neq \Lambda_m$ *for all* $m, k \in \mathbb{N}$ *with* $m \neq k$*;* $\Re(\Lambda_k) > 0$ *for every* $k \geq 1$*;*
- *for some* $\beta > 0$, $|\Im(\Lambda_k)| \leq \beta\sqrt{\Re(\Lambda_k)}$, *for any* $k \geq 1$;
- $|\Lambda_k| \leq |\Lambda_{k+1}|$, *for any* $k \geq 1$*;*
- *Gap condition: for some* $\rho, q > 0$

$$\rho\left|k^2 - m^2\right| \leq |\Lambda_k - \Lambda_m| \ \text{ for any } m, k \geq 1 : |k - m| \geq q;$$

$$\inf_{k\neq m:|k-m|<q} |\Lambda_k - \Lambda_m| > 0$$

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Theorem (continue...)

- *There exist* p_0, p_1, p_2 *with* $p_1, p_2 \geq p_0 > 0$ *such that one has,*

$$-\varpi + p_1\sqrt{r} \leq \mathcal{N}(r) \leq \varpi + p_2\sqrt{r}, \quad \forall r > 0,$$

  where $\mathcal{N}$ *is the counting function associated with the sequence* Λ*, defined by*

$$\mathcal{N}(r) = \#\left\{k : |\Lambda_k| \leq r\right\}, \quad \forall r > 0. \tag{9}$$

Then, there exists $T_0 > 0$ *such that, for every* $\eta \geq 1$ *and* $0 < T < T_0$*, we can find a family of complex valued functions* $\{\Psi_{k,j}\}_{k\geq 1, 0\leq j\leq \eta-1} \in L^2\left(-\frac{T}{2}, \frac{T}{2}\right)$ *biorthogonal to* $\{e_{k,j}\}_{k\geq 1, 0\leq j\leq \eta-1}$*, where for every* $t \in (-\frac{T}{2}, \frac{T}{2})$, $e_{k,j} = t^j e^{-\Lambda_k t}$ *with in addition,*

$$\left\|\Psi_{k,j}\right\|_{L^2(-\frac{T}{2},\frac{T}{2})} \leq Ce^{C\sqrt{\Re(\Lambda_k)}+\frac{C}{T}}. \tag{10}$$

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BOUNDARY CONTROLLABILITY OF THE DEGENERATE SYSTEM IN 2-*d*

Let us consider $(\lambda_{\alpha_2,k}, \phi_{\alpha_2,k})_{k\in\mathbb{N}}$ the corresponding eigenvalues and eigenfunctions of the problem

$$\begin{cases} -(y^{\alpha_2}\phi')'(y) = \lambda\phi(y) \quad y \in (0,1), \\ \begin{cases} \phi(0) = 0 & \text{if } 0 \leq \alpha_2 < 1 \\ (y^{\alpha_2}\phi')(0) = 0 & \text{if } 1 < \alpha_2 < 2 \end{cases}, \\ \phi(1) = 0, \end{cases} \tag{11}$$

Lemma

Any function $u \in H^1_{\alpha,0}(\Omega)$ has the following representation:

$$u = \sum_{j=1}^{\infty} \left\langle u, \phi_{\alpha_2,j} \right\rangle_{L^2(0,1)} \phi_{\alpha_2,j}.$$

Adjoint system

Let us write the adjoint system:

$$
\begin{cases}
\partial_t \sigma + \operatorname{div}(D\sigma) = 0 & \text{in } (0,T)\times\Omega, \\
\sigma = 0 & \text{on } (0,T)\times\partial\Omega \\
\sigma(T) = \sigma_T & \text{in } \Omega,
\end{cases}
\tag{12}
$$

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Partial Observability

Theorem

Let $T > 0$ be given and we assume null control 1-d Theorem holds. Let $C > 0$ be the constant provided and define $C_T := Ce^{C/T}$. Let $0 \leq \alpha_1 < 1$.Then, for all $\sigma_T \in H^1_{\alpha,0}(\Omega)$, we have the following partial observability inequality

$$\left\| \Pi_{E_{\alpha,J}} \sigma(0) \right\|^2_{H^1_{\alpha,0}(\Omega)} \leq (C_T)^2 e^{2C\sqrt{\lambda_{\alpha_2,J}}} \int_0^T \| \mathbf{1}_\gamma (x^{\alpha_1} \sigma_x(t)) \|^2_{L^2(\partial\Omega)} \, \mathrm{d}t \tag{13}$$

where σ is the solution of the two-dimensional adjoint system (12) *with* $\sigma(T) = \sigma_T$.

$$\begin{cases} \partial_t u = \operatorname{div}(D \nabla u) & \text{in } (0,T) \times \Omega, \\ u(t) = 1_\gamma q(t), & \text{in } (0,T) \times \partial\Omega, \\ u(0) = u_0, & \text{in } \Omega. \end{cases} \tag{14}$$

Theorem

Let $T > 0$. There exists $C_T > 0$ such that the following two properties are equivalent

- *For every $u_0 \in E^{-1}_{\alpha,J}$, there exists a control $q \in L^2(0,T;L^2(\partial\Omega))$ such that*
$$\begin{cases} \Pi_{E^{-1}_{\alpha,J}} u(T) = 0 \\ \|q\|_{L^2(0,T;L^2(\partial\Omega))} \leq C_T e^{C\sqrt{\lambda_{\alpha_2,J}}} \|u_0\|_{H^{-1}_{\alpha}(\Omega)}, \end{cases}$$
where u is the solution of the system.
- *For all $\sigma_T \in E_{\alpha,J}$, the solution σ of the adjoint system* (12) *satisfies* (13).

Dissipation along the y-direction

Proposition

Let us consider system (14) *and assume that in some time interval* (t_0, t_1) *control* $q = 0$ *and also assume that for a* $J \geq 1$, $\Pi_{E^{-1}_{\alpha,J}} u(t_0) = 0$. *Then, we have the following estimate*

$$\|u(t)\|_{H^{-1}_{\alpha}(\Omega)} \leq Ce^{-\lambda_{\alpha_2,J+1}(t-t_0)} \|u(t_0)\|_{H^{-1}_{\alpha}(\Omega)}, \forall t \in (t_0, t_1).$$

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Following Steps

Step 1 A time-splitting procedure. $[0,T) = \cup_{k=0}^{\infty}[a_k, a_{k+1}]$

Step 2 Estimate on the interval $[a_k, a_k + T_k]$

Step 3 Estimate on the interval $[a_k + T_k, a_{k+1}]$.

Step 4 Final Estimate.

$$\|u(a_{k+1})\|_{H_\alpha^{-1}(\Omega)} \leq Ce^{-C\beta 2^{k(2-\rho)}} \|u_0\|_{H_\alpha^{-1}(\Omega)} .$$

- Conclusion: we obtain a control function.

$$\|q\|_{L^2(0,T;L^2(\partial\Omega))} = \sum_{k=0}^{\infty} \|q\|_{L^2(a_k, a_k+T_k; L^2(\partial\Omega))}$$

$$\|u(T)\|_{H_\alpha^{-1}(\Omega)} = \lim_{k\mapsto\infty} \|u(a_{k+1})\|_{H_\alpha^{-1}(\Omega)} = 0.$$

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OTHER PROBLEMS

- Can be solved mixing $x^{\alpha_1}, y^{\alpha_2}$ with strong or weak degeneracy.

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- It is an open problem with general degeneracies $a(x), b(y)$.

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- It is an open problem with general degeneracies $a(x), b(y)$.
- It is possible to solve the problem controlling on other sets.

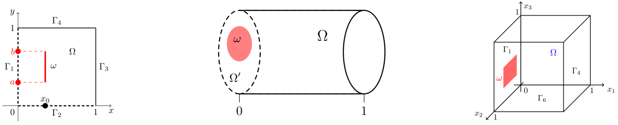

Figure: Different control regions

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- It is an open problem with general degeneracies $a(x), b(y)$.
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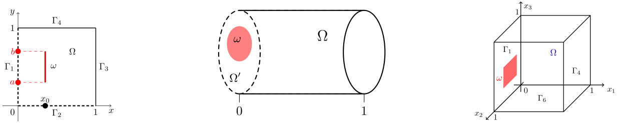

Figure: Different control regions

- It is an open problem to control from the boundary in other n-dimensional geometries.
- The result is valid for coupled degenerate equations.

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¡Felicidades Manolo y Kisko!

¡Bienvenidos al sexto piso!

¡GRACIAS!

THANK YOU!

MERCI!