Workshop on PDEs and Control - PKM60

Institute of Mathematics of the University of Seville (IMUS)

September 3-5, 2025

Book of Abstracts

A conference to celebrate the 60th birthday of Kisko Guillén-González and Manolo González-Burgos

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Workshop on PDEs and Control - PKM60

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Welcome

On behalf of the Organizing Committee, it is a pleasure to welcome all the participants in the Workshop on PDEs and Control, to be held on September from 3 to 5, 2025 at the Institute of Mathematics of the University of Seville.

The development of Partial Differential Equations (PDEs) represents one of the cornerstones of research at the Department of Differential Equations and Numerical Analysis (EDAN) of the University of Seville. Over the years, the department has fostered numerous collaborations, trained many doctoral students, and established a strong network of international contacts, all aimed at advancing research in this fundamental area of mathematics. Thanks to the dedication of its members, EDAN has become a reference point in both research and education in mathematics, while also increasing its level of international engagement. These efforts have led to fruitful collaborations with esteemed researchers from countries such as Brazil, Chile, Colombia, France, Mexico, the USA, Tunisia, and Senegal, promoting a dynamic exchange of ideas and knowledge. With this in mind, EDAN is organizing this international workshop to highlight the latest advances in the field of PDEs and Control, bringing together long-standing collaborators as well as other leading researchers active in these areas. As a special part of the workshop, we will hold a tribute session in honor of Professors Francisco Guillén González and Manuel González Burgos, on the occasion of their 60th birthdays.

The members of the organizing committee wish to express their gratitude to the institutions that have supported and made possible the realization of this event.

It is our hope that this meeting contributes to create a rich and fruitful frame of work and collaboration.

The Organizing Committee

Program

A conference to celebrate the 60th birthday of Kisko Guillén-González and Manolo González-Burgos

Workshop on PDEs and Control 2025 (PKM-60)
Seville, September, 3-5, 2025

Schedule	Wednesday 3	Thursday 4	Friday 5
08:30-09:00	Registration		
09:00-09:45	Opening ceremony	**Plenary 4: Didier Bresch**	**Plenary 8: Assia Benabdallah**
09:45-10:30	**Plenary 1: Enrique Fernández-Cara**	**Plenary 5: Ramon Codina**	**Plenary 9: Farid Ammar Kodja**
10:30-11:15	Coffee break	Coffee break/Posters	Coffee break
11:15-12:00	**Plenary 2: Sylvain Ervedoza**	**Plenary 6: Luz de Teresa**	**Plenary 10: Vivette Giraut**
12:00-12:45	**Plenary 3: Gabriela Planas**	**Plenary 7: Morgan Morancey**	**Plenary 11: Marko Rojas-Medar**
12:45-13:30	About PKM60	Official Photo	Posters
13:30-15:00	Lunch	Lunch	Lunch
15:00-15:30	Giuseppe Viglialoro	Anibal Coronel	Roberto Carlos Cabrales
15:30-16:00	Roman Vanlaere	Exequiel Mallea-Zepeda	Mayte Pérez
16:00-16:30	Mathilda Trabut	Jone Apraiz	Guillaume Olive
16:30-17:00	Roberto Morales	Elder Villamizar	Jérôme Le Rousseau
17:00-17:30	Coffee break	Coffee break	Closing ceremony
17:30-18:00	Imaculada Gayte	Diego Rueda	
18:00-18:30	David Mellado	Pablo Braz e Silva	
18:30-19:00	Daniel Acosta		
19:00-19:30	Everaldo Bonotto		
20:30	Welcome Cocktail		
21:00		Social Dinner at Rio Grande Restaurant	

Acknowledgements

The organizers wish to express their gratitude to several people whose help and assistance have been determinant for the celebration and success of this meeting. Thus, our deep thanks go to IMUS and EDAN secretaries for their help with the administrative issues, specially to Mme Teresa Ayuga. We also are indebted to the Vice-Rectorate of Research of the Universidad de Sevilla, for the continuous support. Finally, we would like to thank EDAN, IMUS, SEMA and US for their financial support.

A conference to celebrate the 60th birthday of
Kisko Guillén-González and Manolo González-Burgos

Workshop on PDEs and Control 2025 (PKM-60)

Seville, September, 3-5, 2025

Speakers:
Farid AMMAR-KHODJA (France)
Assia BENABDALLAH (France)
Didier BRESCH (France)
Ramón CODINA (Spain)
Luz De TERESA (Mexico)
Sylvain ERVEDOZA (France)
Enrique FERNÁNDEZ-CARA (Spain)
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Plenary Talks

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Minimality and control for parabolic systems

F. AMMAR KHODJA*, A. BENABDALLAH**

The aim of this talk is to link the controllability of parabolic problems to the minimality of sequences in a Hilbert space. We will start from the contributions of Manuel González-Burgos (and his collaborators on the boundary control of parabolic systems) to arrive at the recent results that we have obtained with Manuel González-Burgos, Morgan Morancey and Luz de Teresa. The presentation will be in two parts (one by F. Ammar Khodja and the other by A. Benabdallah). Here is the outline :

1. Boundary control of parabolic systems and the moments method : Manolo's contributions;
2. Minimal sequences in Hilbert spaces;
3. Application 1 : Carleman inequalities and minimal sequences;
4. Application 2 : Spectral inequality and minimal sequences;
5. Some extensions and open problems for the union of minimal sequences: applications to simultaneous controllability.

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Minimality and control for parabolic systems

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The aim of this talk is to link the controllability of parabolic problems to the minimality of sequences in a Hilbert space. We will start from the contributions of Manuel González-Burgos (and his collaborators on the boundary control of parabolic systems) to arrive at the recent results that we have obtained with Manuel González-Burgos, Morgan Morancey and Luz de Teresa. The presentation will be in two parts (one by F. Ammar Khodja and the other by A. Benabdallah). Here is the outline :

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Once upon a time Kisco & Manolo

D. Bresch [1]

Once upon a time, there were two young mathematicians that I had the great fortune to meet for the first time when I came for 3 months during my thesis (supervised by Jacques (Simon)) at the invitation of Enrique and Rosa. The first thing that struck me was their complementarity and their great complicity. An efficient and caring tandem, both in the field of mathematical research and in more earthly life. Being there to celebrate their birthdays is a joy for me. Everything will be under control (exact or not) and everything will flow in complete dependence on the rheology of the liquid that we will encounter from September 3rd to 5th. To please my friends, I will center my presentation around fluid mechanics (one of the topic on which Kisco is a specialist) by trying the complicated exercise to present several results that I have obtained with collaborators. I have the dream to talk about "leeks" and more seriously to show that it can be good to gain weight, to show that undergoing constraints sometimes brings beautiful mathematical stories, to show that duality approach could be important to link some phenomena from micro to mesoscales. In this abstract, I would like also to associate Cori (Kisco's wife) and Catherine (my wife) to wish an happy birthday to Kisco and of course to Manolo too. I thank the organizers who came up with the idea for this wonderful conference. Special thanks to Maria Angeles and Anna, long-term collaborators and friends of Kisco and Manolo. I won't list the people who matter to me in Seville because it would be too long. This moment is to celebrate Kisco and Manolo's anniversary. I hope they will enjoy listening to my talk, as well as the other participants. For me, I'm already happy to know that we're going to have a good time.

Acknowledgements

D. Bresch was partially supported by the BOURGEONS project, grant ANR-23-CE40-0014-01 of the French National Research Agency (ANR). This work also benefited of the support of the ANR under France 2030 bearing the reference ANR-23-EXMA-004 (Complexflows project). ;

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Nitsche's prescription of Dirichlet conditions for the conforming finite element approximation of Maxwell's problem

D. Boffi[1], R. Codina[2] and Ö. Türk[3]

This work deals with the FE approximation of the following simplified problem arising in magnetostatics, often called Maxwell's problem: find a magnetic induction field $\boldsymbol{u} : \Omega \longrightarrow \mathbb{R}^d$ and a scalar field $p : \Omega \longrightarrow \mathbb{R}$ solution of the boundary value problem

$$\begin{aligned} \nu \nabla \times \nabla \times \boldsymbol{u} + \nabla p &= \boldsymbol{f} && \text{in } \Omega, \\ -\nabla \cdot \boldsymbol{u} &= 0 && \text{in } \Omega, \\ \boldsymbol{n} \times \boldsymbol{u} &= \boldsymbol{n} \times \bar{\boldsymbol{u}} && \text{on } \Gamma, \\ p &= \bar{p} := 0 && \text{on } \Gamma, \end{aligned}$$

where Ω is a domain of $\mathbb{R}^d$ $(d = 2, 3)$, $\Gamma = \partial\Omega$, $\nu > 0$ is a physical parameter, $\bar{\boldsymbol{u}}$ is given and $\boldsymbol{f}$ is assumed to be solenoidal.

For $\bar{\boldsymbol{u}} = \boldsymbol{0}$, the problem is equivalent to the two variational equations:

$$\begin{aligned} & a(\boldsymbol{u}, \boldsymbol{v}) + b(p, \boldsymbol{v}) = \langle \boldsymbol{f}, \boldsymbol{v} \rangle_\Omega \quad \forall \boldsymbol{v} \in V_0, \\ & b(q, \boldsymbol{u}) = 0 \quad \forall q \in Q_0, \\ & a(\boldsymbol{u}, \boldsymbol{v}) := \nu(\nabla \times \boldsymbol{u}, \nabla \times \boldsymbol{v})_\Omega, \quad b(p, \boldsymbol{v}) := (\nabla p, \boldsymbol{v})_\Omega, \\ & V_0 = H_0(\mathrm{curl}, \Omega), \quad Q_0 = H_0^1(\Omega). \end{aligned}$$

Its conforming finite element approximation consists of building finite element spaces $V_{h,0} \subset V_0$ and $Q_{h,0} \subset Q_0$ and find $\boldsymbol{u}_h \in V_{h,0}, p_h \in Q_{h,0}$ such that

$$\begin{aligned} a(\boldsymbol{u}_h, \boldsymbol{v}_h) + b(p_h, \boldsymbol{v}_h) &= \langle \boldsymbol{f}, \boldsymbol{v}_h \rangle_\Omega && \forall \boldsymbol{v}_h \in V_{h,0}, \\ b(q_h, \boldsymbol{u}_h) &= 0 && \forall q_h \in Q_{h,0}. \end{aligned}$$

When $\bar{\boldsymbol{u}} \neq \boldsymbol{0}$, the boundary condition for $\boldsymbol{u}$ can be prescribed weakly using Nitsche's method, which can also be used to prescribe the boundary condition for p_h. If V_h and Q_h are the finite element spaces without boundary conditions, the problem consists of finding $[\boldsymbol{u}_h, p_h] \in V_h \times Q_h$ such that

$$B_{\mathrm{N}}([\boldsymbol{u}_h, p_h], [\boldsymbol{v}_h, q_h]) = L_{\mathrm{N}}([\boldsymbol{v}_h, q_h]) \quad \forall [\boldsymbol{v}_h, q_h] \in V_h \times Q_h,$$

where

$$\begin{aligned} B_{\mathrm{N}}([\boldsymbol{u}_h, p_h], [\boldsymbol{v}_h, q_h]) &:= \nu(\nabla \times \boldsymbol{v}_h, \nabla \times \boldsymbol{u}_h)_\Omega \\ &+ (\boldsymbol{v}_h, \nabla p_h)_\Omega + (\boldsymbol{u}_h, \nabla q_h)_\Omega \\ &- \nu \langle \boldsymbol{n} \times \boldsymbol{v}_h, \nabla \times \boldsymbol{u}_h \rangle_\Gamma - \langle \boldsymbol{n} \cdot \boldsymbol{u}_h, q_h \rangle_\Gamma \\ &- \nu \langle \boldsymbol{n} \times \boldsymbol{u}_h, \nabla \times \boldsymbol{v}_h \rangle_\Gamma - \langle \boldsymbol{n} \cdot \boldsymbol{v}_h, p_h \rangle_\Gamma \\ &+ N_u \frac{\nu}{h} \langle \boldsymbol{n} \times \boldsymbol{v}_h, \boldsymbol{n} \times \boldsymbol{u}_h \rangle_\Gamma - N_p \frac{L_0^2}{\nu h} (p_h, q_h)_\Gamma \\ L_{\mathrm{N}}([\boldsymbol{v}_h, q_h]) &:= \langle \boldsymbol{v}_h, \boldsymbol{f} \rangle_\Omega - \nu \langle \boldsymbol{n} \times \bar{\boldsymbol{u}}, \nabla \times \boldsymbol{v}_h \rangle_\Gamma \\ &+ N_u \frac{\nu}{h} \langle \boldsymbol{n} \times \boldsymbol{v}_h, \boldsymbol{n} \times \bar{\boldsymbol{u}} \rangle_\Gamma. \end{aligned}$$

We prove that if $V_{h,0}$ and $Q_{h,0}$ satisfy the classical inf-sup condition for Maxwell's problem, then Nitsche's method yields a stable solution that is optimally convergent in the norm:

$$\begin{aligned} \|[\boldsymbol{v}_h, q_h]\|^2_{V \times Q, \mathrm{N}} &:= \nu \|\nabla \times \boldsymbol{u}\|^2_{L^2(\Omega)} + \frac{\nu}{L_0^2} \|\boldsymbol{u}\|^2_{L^2(\Omega)} \\ &+ \frac{L_0^2}{\nu} \|\nabla p\|^2_{L^2(\Omega)} + \frac{\nu}{h} \|\boldsymbol{n} \times \boldsymbol{v}_h\|^2_{L^2(\Gamma)} + \frac{L_0^2}{\nu h} \|q_h\|^2_{L^2(\Gamma)}. \end{aligned}$$

where L_0 is a characteristic length of Ω.

We also prove a similar result for a stabilised finite element method presented in [1], in which spaces $V_{h,0}$ and $Q_{h,0}$ do not need to satisfy any inf-sup condition. The present work is based on [2].

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Boundary null controllability of a class of 2-d degenerate parabolic PDEs

Luz de Teresa[1], Víctor Hernández-Santamaría * and Subrata Majumdar *

In this talk we deal with the boundary null controllability of some degenerate parabolic equations posed on a square domain, presenting, as far as we know, the first study of boundary controllability for such equations in multidimensional settings. The proof combines two classical techniques: the method of moments and a Lebeau-Robbiano strategy. A key novelty of this work lies in the analysis of boundary control localized on a subset of the boundary where degeneracy occurs. Let us consider the following degenerate parabolic equation in a square domain $\Omega = (0,1) \times (0,1)$. We study the degenerate parabolic equation:

$$\begin{cases} \partial_t u = \operatorname{div}(D\nabla u) & \text{in } (0,T) \times \Omega, \\ u(t) = 1_\gamma q(t), & \text{in } (0,T) \times \partial\Omega, \\ u(0) = u_0, & \text{in } \Omega. \end{cases} \tag{1}$$

where $\gamma = \{0\} \times \omega$ with ω an open subset of $(0,1)$ (in the y variable). The matrix function $D : \overline{\Omega} \mapsto M_{2\times 2}(\mathbb{R})$ is given by

$$D(x,y) = \begin{pmatrix} x^{\alpha_1} & 0 \\ 0 & y^{\alpha_2} \end{pmatrix},$$

where $\alpha = (\alpha_1, \alpha_2) \in [0,1) \times [0,1)$, and u_0 is the initial data that lies in a functional space $H_\alpha^{-1}(\Omega)$ We will also give results for $\alpha_1, \alpha_2 \in (1,2)$ and combination of different grades of degeneracy (weak/strong, strong-strong, etc.) [2] We use the ideas in [1] to obtain the boundary control.

Acknowledgements

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On the stabilisation of the incompressible Navier Stokes equations in a 2-d channel with a normal control on the boundary, old and new

Shirshendu Chowdhury[1] and Sylvain Ervedoza[2]

In this talk, I will discuss the stabilization of incompressible Navier-Stokes equations in a 2d channel around a fluid at rest when the control acts only on the normal component of the proper subset of the upper boundary. In this case, the linearized equations are not controllable nor stabilizable at an exponential rate higher than π^2/L^2, when the channel is of width L and of length 2π and the viscosity parameter is set to 1. Our main result allows to go above this threshold and reach any exponential decay rate by using the non-linear term to control the directions which are not controllable for the linearized equations. Our approach therefore relies on writing the controlled trajectory as an expansion of order two taking the form $\varepsilon a + \varepsilon^2 \beta$ for $\varepsilon > 0$ small enough. In particular, we can prove that, for the linearized system, only the 0-mode cannot be controlled and that the other modes are null-controllable when the control acts on the whole upper boundary, and (at least) approximately controllable when the control acts on a localized part of the upper boundary. We thus can develop a non-linear strategy to control a finite number of components of the zero modes through the convective terms of the other modes. This strategy was developed in a prior work with Shirshendu Chowdhury when the non-zero modes are null-controllable. I will explain that it can also be adapted when the non-zero modes are only (known to be) approximately controllable, making the strategy more robust.

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Manolo, Kisko and other young people

E. Fernández-Cara[1]

In this talk, I will first recall some "old" works in collaboration with M. González-Burgos concerning control results for nonlinear PDEs and with F. Guillén, dealing with the mathematical analysis of viscous (but not necessarily Newtonian) fluids.

Then, I will speak of other more recent related results that hopefully will lead to new achievements in the next future. Among others, I will refer to the multi-objective and hierarchic control of several PDEs and also to several geometric inverse problems for parabolic systems.

Acknowledgements

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On a model of flows in a deformable porous solid with small strain and density depending material modulus

A. Bonito[1], D. Guignard[2], V. Girault[3] and K.R. Rajagopal[4]

Consider the following model of incompressible slow flows in a deformable solid in $\mathbb{R}^d$, with an implicit constitutive relation for the Cauchy stress tensor $\mathbf{T}_s$ of the solid:

$$\boldsymbol{\epsilon}_s = E_{1s}(1+\lambda_2 \mathrm{tr}(\boldsymbol{\epsilon}_s))\mathbf{T}_s + E_{2s}(1+\lambda_3 \mathrm{tr}(\boldsymbol{\epsilon}_s))\mathrm{tr}(\mathbf{T}_s)\mathbf{I}, \tag{1}$$

the balance of linear momentum for the solid taking into account the interaction with the fluid through the parameter α:

$$\mathbf{div}(\mathbf{T}_s) + \alpha(\mathbf{v}_f - \partial_t \mathbf{u}_s) = \mathbf{0}, \tag{2}$$

and the flow equation for the fluid taking into account the interaction with the solid:

$$\begin{aligned} \alpha(\mathbf{v}_f - \partial_t \mathbf{u}_s) - \mu_f \Delta \mathbf{v}_f + \nabla p_f &= -\varrho_f \partial_t \mathbf{v}_f, \\ \mathrm{div}\, \mathbf{v}_f &= 0. \end{aligned} \tag{3}$$

Here $\mathrm{tr}(\mathbf{T}_s)$ is the trace of the tensor $\mathbf{T}_s$, $\boldsymbol{\epsilon}_s$ is the symmetric gradient tensor of the solid's displacement $\mathbf{u}_s$, μ_f and ϱ_f are the fluid's viscosity and density and $E_{1s} > 0$ and $E_{2s} < 0$ are elasticity parameters. The system (1)-(2)-(3) is supplemented with initial and boundary conditions.

The model for the solid is an example taken from [1] for small strain, namely

$$\|\boldsymbol{\epsilon}_s\| \leq \delta \ll 1 \tag{4}$$

where $\|\cdot\|$ is the Frobenius norm. In addition a linearized dependence on the density yields the factors $(1 + \lambda_2 \mathrm{tr}(\boldsymbol{\epsilon}_s))$ and $(1 + \lambda_3 \mathrm{tr}(\boldsymbol{\epsilon}_s))$, where λ_2 and λ_3 are also assumed to be small. The resulting relation (1) for $\boldsymbol{\epsilon}_s$ remains nonlinear without compactness nor monotonicity property.

We can take advantage of (4) and suitably truncate $\mathrm{tr}(\boldsymbol{\epsilon}_s)$, i.e., replace $\mathrm{tr}(\boldsymbol{\epsilon}_s)$ by $T_{\tilde{\delta}}\mathrm{tr}(\boldsymbol{\epsilon}_s)$, where T_k is the standard truncation operator at height k and $\tilde{\delta} = \sqrt{d}\delta$. This allows to obtain the following expression for $\mathbf{T}_s$:

$$\mathbf{T}_s = \frac{1}{E_{1s}(1+\lambda_2 T_{\tilde{\delta}} \mathrm{div}\, \mathbf{u}_s)}\Big(\boldsymbol{\epsilon}_s - E_{2s}(1+\lambda_3 T_{\tilde{\delta}} \mathrm{div}\, \mathbf{u}_s)\frac{\mathrm{div}\, \mathbf{u}_s}{F(\mathbf{u}_s)}\mathbf{I}\Big)$$

where

$$F(\mathbf{u}_s) = E_{1s}(1+\lambda_2 T_{\tilde{\delta}} \mathrm{div}\, \mathbf{u}_s) + dE_{2s}(1+\lambda_3 T_{\tilde{\delta}} \mathrm{div}\, \mathbf{u}_s).$$

This new formulation does not change the problem as long as (4) holds. In particular, it does not remedy the lack of compactness and monotonicity but permits to derive some a priori estimates. However, owing that λ_2 and λ_3 are small, the new formulation can now be viewed as a small perturbation of the fully linear model (i.e., with $\lambda_2 = \lambda_3 = 0$) which is itself well-posed. Existence of an exact solution can be obtained by an implicit function argument inspired by [2, 3]. In contrast, the derived a priori estimates allow to directly carry the error analysis of some standard finite element methods without invoking the implicit function theorem.

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Control of parabolic problems and block moment method

M. Morancey[1]

The null controllability of a parabolic problem is equivalent to the resolution of a moment problem. I will start this talk giving this moment problem and briefly recalling its classical resolution using biorthogonal families.

This strategy has proved to be very efficient in some situations where other tools cannot be applied (boundary control of coupled parabolic systems, appearance of a minimal control time). Yet, the use of biorthogonal families does not give optimal results when there is condensation of the eigenvectors of the evolution operator's adjoint.

To overcome this difficulty we introduced with Assia Benabdallah and Franck Boyer the block resolution of moment problems. The goal of this talk is to give an overview of this block moment method and its applications to the study of null controllability for certain parabolic problems in recent years.

I will

- present the method focusing on the case of scalar control problems,
- relate it to known results (Komornik-Loreti, Avdonin-Ivanov) on the hyperbolic setting concerning Riesz bases of divided differences of time exponentials,
- and, if time allows, explain why it is an important tool in the construction of biorthogonal families in higher-dimensional tensorized settings.

This talk is related to different works in collaboration with F. Ammar Khodja, A. Benabdallah, F. Boyer, M. González-Burgos, M. Mehrenberger and L. de Teresa.

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Decay rates to solutions of some dissipative systems in Sobolev critical spaces

Gabriela Planas[1]

Understanding how quickly solutions decay as time approaches infinity is essential for capturing how systems stabilise, how rapidly perturbations vanish, and whether the solutions efficiently reach equilibrium. This understanding provides a link between transient dynamics and the system's long-term behaviour.

In this talk, I will explore the decay rates of solutions in critical Sobolev spaces for a range of dissipative systems. I will present recent results concerning the Navier-Stokes equations, the Navier-Stokes–Coriolis system, the energy-critical nonlinear heat equation, and the Hardy–Sobolev parabolic equation.

The decay estimates are expressed in terms of the decay character of the initial data, yielding algebraic decay rates and showing in detail the roles played by the linear and nonlinear parts. The proof is carried on purely in the critical space. This is the first instance in which such a method is used for obtaining decay bounds in a critical space for nonlinear equations.

In collaboration with M. Ikeda (Japan), L. Kosloff (Brazil), and C.J. Niche (Brazil).

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Some Questions About Second-Grade Fluid Equations

Marko A. Rojas-Medar[1]

In this talk, we will present some results obtained in [1], [3], [4], [5], [6], on the flow of an incompressible non-Newtonian fluid of grade two in $\Omega \times (0,\infty) \subseteq \mathbb{R}^3 \times (0,\infty)$:

(1)
$$\begin{cases} \partial_t(\mathbf{u} - \alpha\Delta\mathbf{u}) - \mu\Delta\mathbf{u} + \mathrm{rot}(\mathbf{u} - \alpha\Delta\mathbf{u}) \times \mathbf{u} + \nabla p = \mathbf{0}, \\ \mathrm{div}\,\mathbf{u} = 0, \\ \mathbf{u}(0) = \mathbf{u}_0. \end{cases}$$

Here, $\mathbf{u}$ and p denote the fluid velocity and pressure, respectively. Furthermore, $\mu > 0$ represents the kinematic viscosity and $\alpha > 0$ is a parameter characterizing the fluid's non-Newtonian behavior.

Next, let us consider the natural norm

$$\|\mathbf{u}(t)\|^2_{H^1_\alpha(\mathbb{R}^3)^3} := \|\mathbf{u}(t)\|^2_{L^2(\mathbb{R}^3)^3} + \alpha\|\nabla\mathbf{u}(t)\|^2_{L^2(\mathbb{R}^3)^3}$$

and the following function space:

$$V_2(\mathbb{R}^3)^3 := \{\mathbf{u} : \mathbf{u} \in H^1(\mathbb{R}^3)^3, \mathrm{curl}(\mathbf{u} - \alpha\Delta\mathbf{u}) \in L^2(\mathbb{R}^3)^3\}.$$

We also define, for $\mathbf{u}_0 \in L^2(\mathbb{R}^n)^n$ and $r \in \left(-\frac{n}{2}, \infty\right)$, the upper decay indicator of $\mathbf{u}_0$ by

$$P_r(\mathbf{u}_0)_+ := \limsup_{\rho\to 0^+} \rho^{-2r-n} \int_{B_\rho} |\widehat{\mathbf{u}_0}(\xi)|^2\, d\xi,$$

where $B_\rho := \{\xi \in \mathbb{R}^n : |\xi| \le \rho\}$. Moreover, we define the upper decay character of $\mathbf{u}_0 \in L^2(\mathbb{R}^n)^n$ by

$$r^*_+(\mathbf{u}_0) := \sup\{r \in \mathbb{R} : P_r(\mathbf{u}_0)_+ < \infty\}.$$

In relation to the article [4], our main results are as follows:

THEOREM 1 *Let $\mathbf{u}_0 \in V_2(\mathbb{R}^3)^3$ and suppose that $r^*_+(\mathbf{u}_0) = r^*_+ \in \left(-\frac{3}{2}, \infty\right)$. Additionally, assume that $\|\mathbf{u}_0\|_{V_2(\mathbb{R}^3)^3} < \epsilon$ for a sufficiently small $\epsilon > 0$. Then, for any weak solution $\mathbf{u}$ to (1), the following estimate holds:*

$$\|\mathbf{u}(t)\|^2_{H^1_\alpha(\mathbb{R}^3)^3} \le C\,(t+1)^{-\min\left\{\frac{3}{2}+r^*_+,\frac{5}{2}\right\}}, \qquad \forall\, t \ge 0,$$

where the constant $C > 0$ depends only on $\|\mathbf{u}_0\|_{V_2(\mathbb{R}^3)^3}$, α, r^*_+, and μ.

We also compare the evolution of solutions $\mathbf{u}(t)$ to (1) with the solutions $\overline{\mathbf{u}}(t)$ of the linear system associated, which is the following pseudo-parabolic equation in $\mathbb{R}^3 \times (0,\infty)$:

(2)
$$\begin{cases} \partial_t(\overline{\mathbf{u}} - \alpha\Delta\overline{\mathbf{u}}) - \mu\Delta\overline{\mathbf{u}} = \mathbf{0}, \\ \mathrm{div}\,\overline{\mathbf{u}} = 0, \\ \overline{\mathbf{u}}(0) = \mathbf{u}_0. \end{cases}$$

THEOREM 2 *Let $\mathbf{u}_0 \in H^4(\mathbb{R}^3)^3$ with $\mathrm{div}\,\mathbf{u}_0 = 0$, and suppose that $\mathbf{u}_0$ is small in $V_2(\mathbb{R}^3)^3$, as in Theorem 1. Let $\mathbf{u}$ be a weak solution to (1), and let $\overline{\mathbf{u}}$ be the solution to the linear part (2) with the same initial data $\mathbf{u}_0 \in H^4(\mathbb{R}^3)^3$. Then, for $r^*_+(\mathbf{u}_0) = r^*_+$, with $-\frac{3}{2} < r^*_+ < \infty$, we have*

$$\|\mathbf{u}(t) - \overline{\mathbf{u}}(t)\|^2_{H^1_\alpha(\mathbb{R}^3)^3} \le C\,(t+1)^{-\min\left\{\frac{5}{2}+\frac{3}{2}r^*_+,\frac{5}{2}\right\}}, \qquad \forall\, t \ge 0,$$

i.e., the solution $\mathbf{u}$ of (1) is asymptotically equivalent to the solution $\overline{\mathbf{u}}$ of the pseudo-parabolic equation (2) with the same data.

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Contributed Talks

Structure-preserving approximation of diffuse interface tumor growth models

D. Acosta-Soba[1], F. Guillén-González[2] and J. R. Rodríguez-Galván[1]

This work is devoted to the numerical approximation of the a degenerate diffuse interface model for tumor growth. This model couples the Cahn-Hilliard equation for the tumor volume fraction u, a reaction-diffusion equation for the nutrient volume fraction n and the Darcy equation for the extracellular fluid velocity $\mathbf{v}$. Let $\Omega \subset \mathbb{R}^d$ be a smooth bounded domain and $T > 0$. The model, introduced in [1], reads as follows: find $(\mathbf{v}, p, u, \mu_u, n)$ with $u, n \in [0, 1]$ such that

$$\mathbf{v} = -\nu K(\nabla p + u\nabla\mu_u + n\nabla\mu_n), \tag{1a}$$

$$\nabla\cdot\mathbf{v} = 0, \tag{1b}$$

$$\partial_t u + \nabla\cdot(u\mathbf{v}) = C_u\nabla\cdot(M(u)\nabla\mu_u) + \delta P_0 P(u,n)(\mu_n-\mu_u)_\oplus, \tag{1c}$$

$$\mu_u = F'(u) - \varepsilon^2\Delta u - \chi_0 n, \tag{1d}$$

$$\partial_t n + \nabla\cdot(n\mathbf{v}) = C_n\nabla\cdot(M(n)\nabla\mu_n) - \delta P_0 P(u,n)(\mu_n-\mu_u)_\oplus, \tag{1e}$$

in $\Omega \times (0,T)$, satisfying $u(0) = u_0$, $n(0) = n_0$ in Ω, with $u_0, n_0 \in L^2(\Omega)$ and $u_0, n_0 \in [0,1]$, and the following boundary conditions on $\partial\Omega \times (0,T)$,

$$\mathbf{v}\cdot\mathbf{n} = \nabla u\cdot\mathbf{n} = (M_n\nabla\mu_n)\cdot\mathbf{n} = (M_u\nabla\mu_u)\cdot\mathbf{n} = 0, \tag{1f}$$

where the parameters above are nonnegative with $\delta, C_u, C_n, K > 0, \varepsilon, \chi_0, P_0 \geq 0$ and $\nu \in \{0,1\}$.

Moreover, $F : \mathbb{R} \to \mathbb{R}$ is the Ginzburg-Landau double-well potential $F(u) = \frac{1}{4}u^2(1-u)^2$, $M(\cdot)$ is a degenerate mobility function, μ_u and μ_n are the chemical potentials for the tumor and nutrient variables, respectively, with $\mu_n := \frac{1}{\delta}n - \chi_0 u$ and $P(\cdot,\cdot)$ is a degenerate proliferation function. The operator $(\cdot)_\oplus$ denotes the positive part such that $(\phi)_\oplus = \max\{\phi, 0\}$.

In this talk, we introduce a structure-preserving discretization of the model (1) based on a upwind discontinuous Galerkin approximation in space and a semi-implicit scheme in time. The resulting discrete variables preserve the following properties of any solution $(\mathbf{v}, p, u, \mu_u, n)$ of the continuous model (1):

Proposition 3 *The total mass of tumor cells and nutrients is conserved in the sense of* $\partial_t \int_\Omega (u(x,t) + n(x,t))dx = 0$.

Proposition 4 *The following energy law is satisfied*

$$\partial_t E(u,n) + C_u\int_\Omega M(u)|\nabla\mu_u|^2 + C_n\int_\Omega M(n)|\nabla\mu_n|^2 + \delta P_0\int_\Omega P(u,n)(\mu_u-\mu_n)_\oplus^2 + \frac{1}{K}\int_\Omega |\mathbf{v}|^2 = 0,$$

where the energy functional is defined by

$$E(u,n) := \int_\Omega \left(\frac{\varepsilon^2}{2}|\nabla u|^2 + F(u) - \chi_0 u\, n + \frac{1}{2\delta}n^2\right).$$

Moreover, we will also present some numerical results that illustrate the performance of the proposed scheme.

The details of the numerical approximation and the proofs of the results in the absence of the extracellular fluid interaction ($\nu = 0$) have been published in [2]. Currently, we are working on extending the ideas for the case $\nu = 1$ using the stabilization techniques developed in [3], direction in which we have made some promising progress that will be reported.

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Attractors for a class of impulsive systems

Everaldo de Mello Bonotto[1]

The theory of impulsive systems describes the evolution of processes where the continuous dynamics are interrupted by abrupt changes of state.

Definition 5 A *semiflow* on X (denoted by (X,π)) is a family of maps $\{\pi(t)\colon t \in \mathbb{R}_+\}$ acting from X to X such that $\pi(0) = I$, $\pi(t+s) = \pi(t)\pi(s)$ for all $t,s \in \mathbb{R}_+$, and $\mathbb{R}_+ \times X \ni (t,x) \mapsto \pi(t)x \in X$ is continuous.

Definition 6 Given a semiflow (X,π), a nonempty closed subset $M \subset X$ is called an *impulsive set* if for each $x \in M$ there exists $\epsilon_x > 0$ such that $\bigcup_{t\in(0,\epsilon_x)}\{\pi(t)x\} \cap M = \emptyset$.

Definition 7 An *impulsive dynamical system* (X,π,M,I) consists of a semiflow (X,π), an impulsive set $M \subset X$ and a continuous function $I\colon M \to X$ called impulsive function.

The *impact function* associated to (X,π,M,I) is given by

$$\phi(x) = \begin{cases} s, & \text{if} \quad \pi(s)x \in M \;\text{ and } \pi(t)x \notin M,\ 0<t<s, \\ \infty, & \text{if} \quad \pi(t)x \notin M \;\text{ for all }\; t>0. \end{cases}$$

The *impulsive positive trajectory* of $x \in X$ in (X,π,M,I) is a map $\tilde{\pi}(\cdot)x\colon J_x \to X$ defined on some interval $J_x \subseteq \mathbb{R}_+$ containing 0, given inductively by the following way: if $\phi(x) = \infty$ then $\tilde{\pi}(t)x = \pi(t)x$ for all $t \in \mathbb{R}_+$. But, if $\phi(x) < \infty$ then we set $x = x_0^+$ and we define $\tilde{\pi}(\cdot)x$ on $[0,\phi(x_0^+)]$ by

$$\tilde{\pi}(t)x = \begin{cases} \pi(t)x_0^+, & \text{if} \quad 0 \leqslant t < \phi(x_0^+), \\ I(\pi(\phi(x_0^+))x_0^+), & \text{if} \quad t = \phi(x_0^+). \end{cases}$$

Now, set $s_0 = \phi(x_0^+)$, $x_1 = \pi(s_0)x_0^+$ and $x_1^+ = I(\pi(s_0)x_0^+)$. Since $s_0 < \infty$, the previous process can go on, but now starting at x_1^+. If $\phi(x_1^+) = \infty$ then we define $\tilde{\pi}(t)x = \pi(t-s_0)x_1^+$ for all $t \geq s_0$. But, if $s_1 = \phi(x_1^+) < \infty$ i.e., $x_2 = \pi(s_1)x_1^+ \in M$ then we define $\tilde{\pi}(\cdot)x$ on $[s_0, s_0+s_1]$ by

$$\tilde{\pi}(t)x = \begin{cases} \pi(t-s_0)x_1^+, & \text{if} \quad s_0 \leqslant t < s_0+s_1, \\ I(x_2), & \text{if} \quad t = s_0+s_1. \end{cases}$$

Here, we denote $x_2^+ = I(x_2)$. This process ends after a finite number of steps if $\phi(x_n^+) = \infty$ for some $n \in \mathbb{N}$, or it may proceed indefinitely, if $\phi(x_n^+) < \infty$ for all $n \in \mathbb{N}$ and, in this case, $\tilde{\pi}(\cdot)x$ is defined in $[0,T(x))$, where $T(x) = \sum_{i=0}^{\infty} s_i$. We shall assume that $T(x) = \infty$ for all $x \in X$.

Definition 8 A nonempty set $\tilde{\mathcal{A}} \subset X$ is called a *global attractor* for (X,π,M,I) if $\tilde{\mathcal{A}}$ is pre-compact and $\tilde{\mathcal{A}} = \overline{\tilde{\mathcal{A}}}\backslash M$, $\tilde{\mathcal{A}}$ is $\tilde{\pi}$-invariant ($\tilde{\pi}(t)A = A$ for all $t \in \mathbb{R}_+$), and $\mathrm{d_H}(\tilde{\pi}(t)B, \tilde{\mathcal{A}}) \overset{n\to\infty}{\longrightarrow} 0$ for every bounded set $B \subset X$, where $\mathrm{d_H}$ is the Hausdorff semidistance.

Let $\hat{X} = \{x \in I(M)\colon \phi(x_k^+) < \infty \text{ for all } k \in \mathbb{N}\}$ and $g\colon \hat{X} \to \hat{X}$ be given by $g(x) = I(\pi(\phi(x))x)$. The system $(\hat{X}, g)$ defines a discrete dynamical system on $\hat{X}$ associated with the impulsive dynamical system (X,π,M,I).

Definition 9 A set $\hat{\mathcal{A}} \subset \hat{X}$ is called a *discrete global attractor* for $(\hat{X},g)$ if $\hat{\mathcal{A}}$ is compact, $\hat{\mathcal{A}}$ is g-invariant ($g(\hat{B}) = \hat{B}$), and $\mathrm{d_H}(g^n(\hat{B}), \hat{\mathcal{A}}) \overset{n\to\infty}{\longrightarrow} 0$ for every bounded set $\hat{B} \subset \hat{X}$.

In this work, we establish sufficient conditions for the existence of global attractors for the systems (X,π,M,I) and $(\hat{X},g)$. Furthermore, we investigate the relationship between these attractors. An application involving a nonlinear reaction-diffusion initial boundary value problem is also presented.

Acknowledgements

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Eigenvalue bounds for micropolar shear flows

P. Braz e Silva[1]

Linear stability for general viscous 2D micropolar shear flows [3]

$$\mathbf{U} = (U(y), 0, 0),\ \mathbf{W} = (0, 0, W(y)),\ y \in (0, 1),$$

is determined by the (dimensionless) equations [2]

$$\begin{aligned}
&i\alpha \left[(U-c)(D^2-\alpha^2) - U''\right] \widetilde{\psi} = \\
&\quad = \left(\frac{1}{R_\mu} + \frac{1}{2R_k}\right)(D^2-\alpha^2)^2\widetilde{\psi} - \frac{R_0}{R_k}(D^2-\alpha^2)\widetilde{w}, \\
&i\alpha \left[(U-c)\widetilde{w} - W'\widetilde{\psi}\right] = \\
&\quad = \frac{1}{R_\gamma}(D^2-\alpha^2)\widetilde{w} - \frac{2R_0}{R_\nu}\widetilde{w} + \frac{1}{R_\nu}(D^2-\alpha^2)\widetilde{\psi},
\end{aligned}$$

where R_γ, R_μ, R_ν, R_k, and R_0 are dimensionless parameters and $D := \frac{d}{dy}$. Let $c = c_r + ic_i$ be any eigenvalue of this system. We show bounds for both its real and imaginary parts. The bounds obtained for the imaginary part c_i assure linear stability for the flow in an specific region of the parameters of the problem. These bounds are analogous to the classical result of [1] for flows governed by the Navier-Stokes equations, thus generalizing this classical result to the micropolar case. These results were published in [4].

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Non-residual-based stabilization formulation for liquid-solid phase-change flows including macrosegregation scenarios

R.C. Cabrales[1], E. Castillo[2] and R. Codina*[3]

We present a variational multiescale (VMS) finite element method algorithm to solve liquid-to-solid phase-change problems, including macrosegregation. If $\nabla^s \boldsymbol{u} = \frac{1}{2}(\nabla \boldsymbol{u} + [\nabla \boldsymbol{u}]^T)$ is the symmetrized velocity gradient, the conservation governing equations solved in $Q = \Omega \times \Upsilon$ are the linear momentum, the continuity, the energy equation and the concentration of species given by:

$$\rho \frac{\partial \boldsymbol{u}}{\partial t} + \rho(\boldsymbol{u} \cdot \nabla)\boldsymbol{u} - \nabla \cdot (2\mu \nabla^s \boldsymbol{u}) + \nabla p + \boldsymbol{\mathcal{K}}_\varepsilon(f_s, \boldsymbol{u}) = \boldsymbol{f},$$

$$\nabla \cdot \boldsymbol{u} = 0,$$

$$\rho C_p \frac{\partial T}{\partial t} + \rho C_p \boldsymbol{u} \cdot \nabla T - \kappa \Delta T = \rho L \frac{\partial f_s}{\partial t}$$

$$\frac{\partial c}{\partial t} + \boldsymbol{u} \cdot \nabla c_l = 0,$$

with $\boldsymbol{f} = \rho \boldsymbol{g}\,(1 - \beta_T(T - T_r) - \beta_c(c_l - c_r))$ the volumetric force term modeling the coupling between the linear momentum equation and the temperature and concentration fields through a double diffusion mechanism, including natural heat and mass convection. The liquid-solid phase change phenomena is described with a modified Carman-Kozeny model:

$$\boldsymbol{\mathcal{K}}_\varepsilon(f_s, \boldsymbol{u}) = \frac{C_0 \mu f_s^2}{\lambda^2[(1 - f_s)^3 + \varepsilon]}\boldsymbol{u},$$

with $C_0 > 0$, λ the interdendritic space, and the numerical parameter $\varepsilon > 0$ used to avoid numerical singularity when $f_s = 1$. The liquid concentration c_l is calculated by the equation

$$c_l = \frac{c}{1 - (1 - r)f_s},$$

and the solid fraction f_s by using the phase diagram and the level rule.

The VMS framework allows equal-order interpolation for all variables and convective dominant scenarios. The formulation has a dynamic term-by-term structure, which reduces the number of stabilization terms to the minimum, ensuring optimal order of convergence of the solved fields [1]. Since the problem involves coupling of the velocity, pressure, temperature and species concentration fields, the resolution algorithm of the equations is important. This work proposes an orthogonal and dynamic term-by-term stabilization to approximate numerically liquid-solid phase change flows, including macrosegregation scenarios [4]. We present convergence test to highlight the optimal convergence order of the stabilized formulation used, the validation of the method with classical numerical and experimental benchmarks, and the approximation of a Pb-Sn binary alloy problem that generates convection plumes.

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The optimal control approach to analyze some inverse problems for reaction-diffusion systems arising from epidemiology

A. Coronel[1], F. Huancas[2] and M. Sepulveda[3]

In this talk, we consider the parameter identification problem for two kind of reaction-diffusion systems: the compartmental model susceptible-infected–susceptible (SIS) with diffusion. The direct problem that we consider is a susceptible-infected-susceptible mathematical model with cross-diffusion, which was deduced by assuming the following hypotheses: The total population can be partitioned into susceptible and infected individuals; a healthy susceptible individual becomes infected through contact with an infected individual; there is no immunity, and infected individuals can become susceptible again; the spread of epidemics arises in a spatially heterogeneous environment; the susceptible and infected individuals implement strategies to avoid each other by staying away. The spread of the dynamics is governed by an initial boundary value problem for a reaction-diffusion system, where the model unknowns are the densities of susceptible and infected individuals, and the boundary condition models the fact that there is neither emigration nor immigration through their boundary. The reaction consists of two terms: modeling disease transmission and infection recovery and the diffusion cross-diffusion matrix arising from the assumption that the motion of susceptibles is affected by taxis. The inverse problem is the determination of rates on the reaction and diffusion terms from observation of an infected profile on a fixed time. We reformulated the identification problem as an optimal control problem where the cost function is a regularized least squares function. The fundamental contributions of this article are the following: The existence of at least one solution to the optimization problem or, equivalently, the diffusion identification problem; the introduction of first-order necessary optimality conditions; and the necessary conditions that imply a local uniqueness result of the inverse problem. Moreover, we present some numerical results and the extension of the methodology to reaction-diffusion systems arising from oncology.

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Exact controllability to zero for general linear parabolic equations

I. Gayte[1] and I. Marín-Gayte[2]

This paper studies the existence and characterization of partially distributed controls that drive the solution of a linear parabolic problem with general diffusion coefficients to zero in a fixed time T. Specifically, given $\Omega \subset \mathbb{R}^N$ a bounded open set, whose boundary is $C^{0,1}$, $\omega \subset \Omega$ an open set and 1_ω the characteristic function on ω, $\Sigma = \partial\Omega \times (0, T)$ and a matrix $A \in L^\infty(\Omega \times (0, T))^{N\times N}$ satisfying

$$\alpha|\xi|^2 \leq \sum_{i,j=1}^{N} A_{ij}(t, x)\xi_i\xi_j \leq \beta|\xi|^2 \quad \forall \xi \in \mathbb{R}^N,$$

we prove that there exists $\hat{u} \in L^2(\Omega \times (0, T))$ such that the solution of the linear parabolic problem

$$\text{(1)} \qquad \begin{cases} \hat{y}_t - \nabla \cdot (A\nabla \hat{y}) = \hat{u}1_\omega \text{ in } \Omega \times (0, T) \\ \hat{y}|_\Sigma = 0 \\ \hat{y}(0) = y_0 \text{ in } \Omega, \end{cases}$$

verifies

$$\hat{y}(T) = 0 \text{ in } \Omega.$$

The result represents a novelty in control theory for two reasons: first, because the elliptic operator given by the matrix A does not have regular coefficients. And the second reason is that domains of class $C^{0,1}$ are sufficient.

In [4] we proved the following theorem:

Theorem 10 *Let be $u \in L^2(\omega \times (0,T))$, $u \geq c > 0$ in $\omega \times (0,T)$ and zero outside of ω, $y_0 \in L^2(\Omega)$, $y_0 \geq 0$. Then, there exists $v^* \in L^2(\omega \times (0,T))$, $0 \leq v^* \leq u$, $\|\Psi_{v^*}(T)\| < \|y(T)\|$ such that*

$$\hat{u} = \frac{\|y(T)\|v^* - \|\Psi_{v^*}(T)\|u}{\|y(T)\| - \|\Psi_{v^*}(T)\|}$$

is a control in ω for the initial data y_0.

The solution of a problem like (1) with right-hand side v^* is denoted by Ψ_{v^*} and $\| \cdot \|$ is the norm in $L^2(\Omega)$.

When the initial data is any function $y_0 \in L^2(\Omega)$, we apply this theorem with y_0^+ and y_0^-.

The proof is based on a kind of maximum principle in the final time and the linearity of the equation. It does not use the standard techniques of Carleman's inequalities, (see [2], [1]), because of the diffusion coefficients are not continuous functions in general. Besides, the spatial dimension is any (see [3]).
An interesting application of control to this type of problem is the diffusion of cancer cells in a brain tumor (see [5]). The model consists of a linear parabolic problem where the diffusion coefficients are piecewise constant functions.

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Geometric inverse problem of determining multidimensional domains

A. Jone Apraiz[1], B. Anna Doubova[2], C. Enrique Fernández-Cara[3] and D. Masahiro Yamamoto[4]

Last decades, the analysis and solution of inverse problems has increased a lot because of their importance in many applications: elastography and medical imaging, seismology, potential theory, ion transport problems or chromatography and other similar fields.

In this talk we will consider a geometric inverse problem for some linear parabolic systems, where the initial data (and even the coefficients) are unknown and the non-homogeneous part of the equation is expressed as a function of separate space and time variables. The aim of the work presented here will be the identification of a subdomain within a multidimensional set.

We will show the results we have obtained for the uniqueness property by incorporating observations that can be on the boundary or in an interior domain. Through this process, we will also see that the information about the initial data can be derived.

During the talk, it will be seen that the main tools that are required for the proofs of these results include unique continuation, time analyticity of the solutions and semigroup theory.

All the work that will be shown in this talk has been written in a preprint and submitted for publication, [1]. Other interesting related works are [2], [3], [4], [5] and [6].

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Boundary estimation for the Stokes system

Jérôme Le Rousseau [1]

Neglecting the inertial term in the Navier–Stokes system leads to the Stokes system

$$\begin{cases} \partial_t U - \Delta U + \nabla q = F, \\ \operatorname{div} U = 0. \end{cases}$$

We are interested in observing this system from an interior region of a domain. We consider general boundary conditions that include, for instance, the commonly used Dirichlet, Navier, and Neumann conditions.

Observation is achieved through a *local Carleman inequality* near a boundary point. This inequality is derived from the full system, including the pressure term.

Carleman inequalities are weighted estimates where the weight is exponential. For the Laplace operator, it takes the form

$$\tau^{3/2} \|e^{\tau\varphi} v\|_{L^2} + \tau^{1/2} \|e^{\tau\varphi} \nabla v\|_{L^2} \leq C \|e^{\tau\varphi} \Delta v\|_{L^2}, \tag{1}$$

for a function v compactly supported. Here, the parameter $\tau > 0$ can be chosen as large as needed. The choice of the function φ is delicate and depends on the intended application of the estimate. For the Laplace operator, this is a *sub-elliptic estimate* with a loss of half a derivative, which is reflected by the term $\tau^{3/2}$ instead of τ^2 in front of the L^2-norm of v on the left-hand side of (1); see for instance [2]. Boundary conditions are also necessary to obtain an estimate as in (1) near the boundary.

For a parabolic operator, the weight function can be chosen as in the seminal work of Fursikov and Imanuvilov [1], that is, $\varphi(t, x) = t^{-1}(T - t)^{-1}\phi(x)$ and the estimation has the form

$$\tau^3 \int_0^T \|\varphi^{3/2} e^{\tau\varphi} v\|_{L^2}^2 dt + \tau \int_0^T \|\varphi^{1/2} e^{\tau\varphi} \nabla v\|_{L^2}^2 dt \leq C \int_0^T \|e^{\tau\varphi} (\partial_t - \Delta) v\|_{L^2}^2 dt. \tag{2}$$

We begin by reviewing how boundary estimates can be obtained for first-order scalar operators. Then, by expressing the Stokes system in the form of a first-order system, we show how various scalar reductions of the Stokes system can lead to such first-order equations using eigenvectors and generalized eigenvectors. This analysis is carried out *microlocally* in different regions of phase space.

Boundary conditions are handled in the spirit of the treatment of Lopatinskiĭ-Šapiro conditions for the Laplace operator; see for instance [3].

We obtain an estimate where there is a loss of a full derivative for the velocity U, and a loss of half a derivative for the pression q. The loss of a full derivative makes the analysis sometimes intricate since this is a threshold that may prevent one to handle remainder terms that appear along some of the estimations. With a weight function as for (2), the final estimation we obtain is of the form

$$\tau^2 \int_0^T \|\varphi e^{\tau\varphi} U\|_{L^2}^2 dt + \int_0^T \|e^{\tau\varphi} \nabla U\|_{L^2}^2 dt + \tau \int_0^T \|\varphi^{1/2} e^{\tau\varphi} q\|_{L^2}^2 dt \leq C \int_0^T \|e^{\tau\varphi} F\|_{L^2}^2 dt, \tag{3}$$

with additional estimations of the Dirichlet and Neumann traces of U and q.

This is joint work with Luc Robbiano (Université de Versailles Saint-Quentin).

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Optimal control problems related to chemo-repulsion systems

F. Guillén-González[1], E. Mallea-Zepeda[2], M.A. Rodríguez-Bellido* and E.J. Villamizar-Roa[3]

Abstract

In this talk we present bilinear optimal control problems related to chemo-repulsion systems with linear and superlinear production terms in the 2D case and linear in the 3D case. We establish results on existence of global optimal solutions and derive the respective optimality systems, based on a result of the existence of Lagrange multipliers in Banach spaces. Finally, we analyze the main differences (and difficulties) between the 2D and 3D cases.

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Stability of nonlinear Dirac solitons under the action of external potentials

D. Mellado-Alcedo[1] and N. R. Quintero[2]

The nonlinear Dirac equation in 1+1-dimensions supports localized solitons. Theoretically, these traveling waves propagate with constant velocity, energy, momentum, and charge. However, the soliton profiles can be distorted, and eventually destroyed, due to intrinsic or numerical instabilities. The constants of motion and the initial profiles can also be modified by external potentials, which may give rise to instabilities.

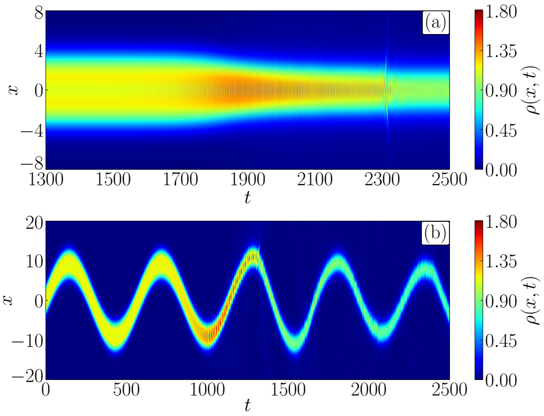

Figure 1: Evolution of the density of the charge from simulations of the ABS model (a) without and (b) with harmonic potential.

In this work [1], we study the instabilities observed in numerical simulations of the Gross-Neveu equation [2] under linear and harmonic potentials. We perform an algorithm [3] based on the method of characteristics to numerically obtain the two soliton spinor components. All studied solitons are numerically stable, except the low-frequency solitons oscillating in the harmonic potential over long periods of time. These instabilities are identified by the non-conservation of both energy and charge, and can be removed by imposing absorbing boundary conditions. We find that the dynamics of the soliton is in perfect agreement with the prediction obtained using an Ansatz with only two collective coordinates. By applying the same methodology, we also demonstrate the spurious character of the reported instabilities in the Alexeeva–Barashenkov–Saxena (ABS) model [4] under external potentials.

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On the local null controllability of a cubic Ginzburg-Landau equation with dynamic boundary conditions

Nicolás Carreño[1], Alberto Mercado* and **Roberto Morales**[2]

Let $\Omega \subset \mathbb{R}^d$ $(d \geq 2)$ be a bounded domain with boundary $\Gamma := \partial\Omega$ of class C^2. For $T > 0$, we define the sets $Q := \Omega \times (0,T)$, $\Sigma := \Gamma \times (0,T)$ and for any subset $\omega \subset \Omega$, we write $Q_\omega := \omega \times (0,T)$.

For $a, b > 0$ and $\alpha \in \mathbb{R}$, we define the linear operators

$$\begin{aligned} L(u) &:= \partial_t u - a(1+\alpha i)\Delta u \\ L_\Gamma(u, u_\Gamma) &:= \partial_t u_\Gamma + a(1+\alpha i)\partial_\nu u - b(1+\alpha i)\Delta_\Gamma u_\Gamma \end{aligned}$$

Moreover, for $c, \gamma \in \mathbb{R}$ we put $f(w) := c(1+\gamma i)|w|^2 w_\Gamma$. Then, we consider the following system

$$\begin{cases} L(u) + f(u) = \mathbb{1}_\omega h & \text{in } Q, \\ L_\Gamma(u, u_\Gamma) + f(u_\Gamma) = 0 & \text{on } \Sigma, \\ u\big|_\Gamma = u_\Gamma & \text{on } \Sigma, \\ (u(\cdot,0), u_\Gamma(\cdot,0)) = (u_0, u_{\Gamma,0}) & \text{in } \Omega \times \Gamma. \end{cases} \tag{1}$$

The control h acts only on the first equation. This means that the second equation (i.e., the general dynamic boundary condition) is controlled by the side condition $u\big|_\Gamma = u_\Gamma$.

Let us define the spaces

$$\begin{aligned} \mathbb{L}^2 &:= L^2(\Omega) \times L^2(\Gamma), \\ \mathbb{H}^k &:= \{(y, y_\Gamma) \in H^k(\Omega) \times H^k(\Gamma) \,:\, y\big|_\Gamma = y_\Gamma\} \end{aligned}$$

It is not difficult to see that, under a smallness condition on both the initial data and the control, a unique solution of (1) exists.

Our main result is the following

Theorem 1.1 *Suppose that* $d = 2$ *or* $d = 3$ *and* $\omega \Subset \Omega$. *Then, the system* (1) *is* **locally null controllable at every time** T *in* $\mathbb{H}^1$, *i.e., there exists* $\delta > 0$ *such that, for every* $(u_0, u_{\Gamma,0}) \in \mathbb{H}^1$ *verifying*

$$\|(u_0, u_{\Gamma,0})\|_{\mathbb{H}^1} \leq \delta,$$

there exists a control $h \in L^2(\omega \times (0,T))$ *such that the solution* (u, u_Γ) *of* (1) *satisfies*

$$u(\cdot, T) = 0 \text{ in } \Omega, \quad u_\Gamma(\cdot, T) = 0 \text{ on } \Gamma.$$

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The minimal control time for the exact controllability by internal controls of 1D linear hyperbolic systems

G. Olive[1]

In this talk we will present the minimal control time for the exact controllability by internal controls of one-dimensional (1D) linear hyperbolic systems when the number of controls is equal to the number of state variables. The controls are supported in space in an arbitrary open subset. This presentation will be based on the work [1] in collaboration with Long Hu (Shandong University), with preliminary material taken from [2, 3], by the same authors. An important ingredient is a technique introduced by Manuel González-Burgos and his collaborators in the survey [4]

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A new mathematical model for cell motility with nonlocal repulsion from saturated areas

Carlo Giambiagi Ferrari [1], Francisco Guillén-González[2], Mayte Pérez-Llanos[3] and Antonio Suárez [4]

The main purpose of this work is the mathematical modelling of large populations of cells under different deterministic interactions among themselves, in balance with random diffusion. We focus on cell-cell interaction mechanisms for a single population confined to an isolated domain. We derive a macroscopic mathematical model including a nonlocal saturation coefficient depending on a crowding capacity, as part of a nonlocal drift term. Then, this capacity acts as a threshold above which repulsion effects appear. This macroscopic model is approached using two different microscopic discrete models based on Eulerian or Lagrangian reference systems, respectively.

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Numerical analysis for a diffusive SIS epidemic model with repulsive infected-taxis

Francisco Guillén-González[1], Diego A. Rueda-Gómez [2] and María A. Rodríguez-Bellido*

Abstract

This talk is devoted to the numerical study of a reaction-diffusion SIS epidemic model with repulsive infected-taxis. This model describes the dynamics of a population, in which susceptible people v may want to stay away from infective one u. By using a regularization technique, we propose a finite element fully discrete scheme using a nonlinear discrete diffusion, which preserves some qualitative properties such as well-posedness, conservation of the total mass, point-wise and uniform estimates for u, positivity for u and approximated positivity for v. The key point to deduce the approximated positivity property, crucial to avoid the appearance of spurious oscillations, is to obtain a discrete estimate of a singular functional associated to infected individuals. Finally, in the course of some numerical simulations, the new scheme performs better than other more classical finite element schemes.

Acknowledgements

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Observability inequality for the Grushin equation on a multi-dimensional domain

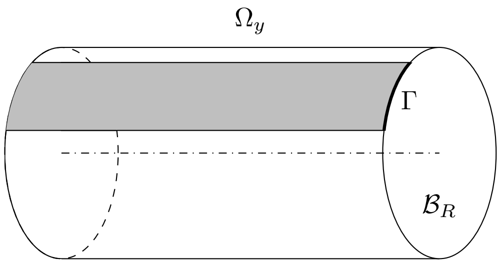
Mathilda Trabut[1]

Introduction and main result

Let $\mathcal{B}_R$ denote the open ball of radius $R > 0$ of $\mathbb{R}^{d_1}$, and Ω_y denote a non-empty bounded open set of $\mathbb{R}^{d_2}$ with $d_1, d_2 > 0$. For $T > 0$, we consider u a solution of the Grushin equation posed in $(0,T) \times \mathcal{B}_R \times \Omega_y$, i.e. satisfying

$$(G) \quad \begin{cases} \partial_t u - \Delta_x u - ||x||^2 \Delta_y u = 0 \text{ in } (0,T) \times \mathcal{B}_R \times \Omega_y, \\ u = 0, \text{ on } (0,T) \times \partial(\mathcal{B}_R \times \Omega_y). \end{cases}$$

It is a degenerate parabolic equation where the diffusion coefficient in y vanishes at the center of the ball.



For $\Gamma \subset \partial\mathcal{B}_R$, we say that the Grushin equation is observable through $\Gamma \times \Omega_y$ at time T if there exists a constant C such that any solution of (G) satisfies

$$(O_T) \quad \int_{\mathcal{B}_R \times \Omega_y} |\nabla u(T)|^2 dx dy \le C \int_0^T \int_{\Omega_y} \int_\Gamma \left| \frac{\partial u}{\partial n} \right|^2 d\sigma(x) dy dt,$$

Theorem 12 *Let Γ be a non-empty open subset of $\partial\mathcal{B}_R$, and $T^* = \frac{R^2}{2d_1}$, then (O_T) holds for all $T > T^*$, and (O_T) does not hold for all $T < T^*$.*

Remarks: The minimal time T^* appears because of the degeneracy at $x = 0$. Indeed if there is no degeneracy, the system reduces to the heat equation which is observable at any $T > 0$.

If $\Gamma = \partial\mathcal{B}_R$, this result is known, see [1]. The goal of this work is to generalize this result if Γ is a non-empty open subset of $\partial\mathcal{B}_R$. Since the negative result on $\partial\mathcal{B}_R$ implies the negative result on Γ, it remains to prove that (O_T) holds for $T > T^*$.

Strategy of proof

We denote $(\lambda_p, \phi_p)_{p\in\mathbb{N}}$ the sequence of eigenvalues and eigenfunctions of the Dirichlet-Laplacian on Ω_y. As it is done in [1] we decompose the equation in the basis $(\phi_p)_{p\in\mathbb{N}}$. Then $u_p := \langle u, \phi_p \rangle$ satisfies the harmonic-heat equation

$$(H_p) \quad \begin{cases} \partial_t u_p - \Delta u_p + \lambda_p^2 ||x||^2 u_p = 0 \text{ in } (0,T) \times \mathcal{B}_R, \\ u_p = 0, \text{ on } (0,T) \times \partial\mathcal{B}_R. \end{cases}$$

The strategy is to prove that (H_p) is observable through Γ uniformly in p. Here are the main steps to get this observability inequality.

- From [2] we already know that (H_p) is observable through $\partial\mathcal{B}_R$ at any time $T > 0$. However, we need to refine this result to get an observability constant which is explicit in T and λ_p.
- We apply a Lebeau-Robbiano type strategy to obtain the observability through Γ. To this end we adapt Miller's proof [3], tracking the dependance in λ_p in the estimates.
- We deduce that (H_p) is observable through Γ uniformly in p at time T, if $T > T^*$.

Once we have the observability inequality for (H_p) with the constant uniform in p, we obtain (O_T) by summing in p.

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On the null-controllability of subelliptic systems of Grushin type

Vanlaere Roman [1]

In this talk, we will discuss the null-controllability properties of the heat equation associated with Grushin-type operators, posed on tensorized domains of two dimensions. That is, parabolic equations associated with second-order operators whose coefficients can be either singular and/or degenerated. Our parabolic operator of interest takes the form

$$\partial_t - \partial_x^2 - q(x)^2 \partial_y (r(y)^2 \partial_y) + V(x) \tag{1}$$

where, q a non-zero function that can vanish, r a strictly positive function, and V is a potential that depends on the choice of a measure on the domain, eventually singular.

We will first discuss negative results, that may be obtained via Agmon theory. Then, we will talk about positive results, either using a constructive approach, as the moments method, or by means of Carleman estimates.

Finally, if time lets us, we shall briefly discuss how these results extend on two-dimensional almost-Riemannian manifolds, for which the metric is degenerated along a submanifold of codimension one.

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On Keller–Segel models with positive total flux: analytic and modeling perspectives

Giuseppe Viglialoro[1]

Since the introduction of the seminal Keller–Segel models, which provide a mathematical framework for describing chemotaxis—the movement of cells or organisms in response to chemical gradients—there has been an explosion of research on various extensions and modifications of these models. These studies have contributed significantly to our understanding of pattern formation, aggregation, and the dynamics of cell populations under the influence of chemical signals. A unifying feature across the majority of this body of work is the imposition of zero-flux (Neumann-type) boundary conditions on the cell density equation, effectively modeling impenetrable domain boundaries that prevent cell escape or entry.

In this talk, we depart from this standard assumption and explore chemotaxis models under the novel premise that the total cell flux across the boundary is strictly positive, thereby modeling domains that are penetrable. Such a setting introduces new analytical challenges and potential biological interpretations, especially in contexts where cell inflow or outflow plays a significant role. We will present some preliminary results in this direction and discuss open questions and considerations that arise in studying these models.

This work is part of an ongoing collaboration with Khadijeh Baghaei, Silvia Frassu and Yuya Tanaka.

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An optimal control problem related to a 3D chemo-repulsion model with nonlinear production

F. Guillén-González [1], M.A. Rodríguez-Bellido [2], E. Mallea-Zepeda[3] and E.J. Villamizar-Roa [4]*

Chemotaxis corresponds to the directional movement of cells or living organisms influenced by the concentration of the chemical signal substance. This motion can be towards a higher (attractive) or lower (repulsive) concentration of the chemical stimuli. In this paper we are interested in the repulsive chemotaxis scenario, in which the presence of living organisms produce chemical substance, including or not a logistic growth of organisms. Explicitly, considering $Q := (0,T) \times \Omega$, with $\Omega \subset \mathbb{R}^3$ being a bounded domain and $(0,T), T > 0$, a time interval, we consider the following system

$$\text{(1)} \quad \left\{ \begin{array}{rcl} \partial_t u - \Delta u &=& \nabla \cdot (u \nabla v) + ru - \mu u^p, \\ \partial_t v - \Delta v + v &=& u^p + f v \, 1_{\Omega_c}, \end{array} \right.$$

where the unknowns are $u(t,x) \geq 0$ and $v(t,x) \geq 0$ denoting the cell density of some chemotactically active species and the chemical concentration. The nonlinear term $\nabla \cdot (u \nabla v)$, on the right-hand-side (1), describes the repulsion mechanism. In addition, $1 \leq p < +\infty$, $r, \mu \geq 0$, and $f = f(t,x)$ is the control function acting on a subdomain $Q_c = (0,T) \times \Omega_c \subset Q = (0,T) \times \Omega$. This system is endowed with initial and non-flux boundary conditions. We prove the existence of global weak solutions when $f \in L^{5/2}(Q_c)$, analyzing the role of the diffusion and the logistic terms to get energy estimates.

Definition 13 (Weak solutions) Let $f \in L^{5/2}(Q_c) := L^{5/2}(L^{5/2}(\Omega_c))$, $(u_0, v_0) \in L^p(\Omega) \times H^1(\Omega)$, with $u_0 \geq 0$ and $v_0 \geq 0$ a.e. in Ω. A pair (u,v) is called weak solution in $[0,T]$ of system (1) with initial data (u_0, v_0) if $u \geq 0$ and $v \geq 0$ a.e. in Q, $u \in L^\infty(L^p) \cap L^{5p/3}(Q)$, $\quad v \in L^\infty(H^1) \cap L^2(H^2)$, and $u \in L^{2p-1}$ for $p > 3$ in the logistic case ($\mu > 0$), and $\nabla u \in L^{\gamma(p)}(Q)$ with

$$\gamma(p) = \left\{ \begin{array}{ll} 5p/(3+p) & \text{when } 1 < p \leq 2, \\ 25p/(18+5p) & \text{when } 2 < p < 12/5, \\ 2 & \text{when } p \geq 12/5, \end{array} \right.$$

satisfying the u-equation $(1)_1$ in a variational sense, and the v-equation $(1)_1$ holds a.e $(t,x) \in Q$.

Theorem 14 (Existence of weak solutions) *Assume that $f \in L^{5/2}(Q_c)$, $(u_0, v_0) \in L^p(\Omega) \times H^1(\Omega)$, with $u_0 \geq 0$ and $v_0 \geq$ a.e. in Ω. If $1 < p \leq 5/3$, then there exists a weak solution of system (1) with $\mu = r = 0$, and if $p > 1$, then there exists a weak solution of system (1).*

Knowing the existence of global weak solutions, we establish a regularity criterion through which weak solutions of systems become strong solutions. These strong solutions will give the adequate framework to study the following optimal control problem:

$$\text{(2)} \quad \left\{ \begin{array}{l} \min J(u,v,f), \\ \text{subject to } (u,v,f) \in \mathcal{S}_{ad}, \end{array} \right.$$

where $J : L^{5p/2}(Q) \times L^2(Q) \times L^{5/2}(L^{5/2+}(\Omega_c)) \to \mathbb{R}$ is the cost functional defined by

$$J = \frac{\gamma_u}{5p/2} \int_0^T \|u - u_d\|_{L^{5p/2}}^{5p/2} dt + \frac{\gamma_v}{2} \int_0^T \|v - v_d\|^2 dt + \frac{\gamma_f}{5/2} \int_0^T \|f\|_{L^{5/2+}(\Omega_c)}^{5/2} dt,$$

for some desired states (u_d, v_d). We prove the existence of global optimal solutions and derive first-order necessary optimality conditions for local optimal solutions. All the results presented here were obtained in [1].

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Posters

Existence of non-autonomous exponential attractors for a reaction-diffusion model with terms in H^{-1}

Aguilar-Reyes, Álvaro[1], Marín-Rubio, Pedro*

In this poster we present the results obtained in [1], where we ensure the existence of pullback exponential attractors for the process associated to a non-autonomous reaction-diffusion problem under minimal regularity assumptions, in fact with H^{-1}-valued time-dependent forces. We recall below the definition of pullback exponential attractor.

Definition 15 Let $\{U(t,s) : t \geq s\}$ be a process in a metric space X. The family $\mathcal{M} = \{\mathcal{M}(t) : t \in \mathbb{R}\}$ is a pullback exponential attractor for the process $\{U(t,s) : t \geq s\}$ in X if:

1. The subsets $\mathcal{M}(t) \subset X$ are non-empty and compact in X for all $t \in \mathbb{R}$.
2. The family is positively semi-invariant, i.e. $U(t,s)\mathcal{M}(s) \subset \mathcal{M}(t)$ for all $t \geq s$.
3. The fractal dimension in X of the sections $\mathcal{M}(t)$ for any $t \in \mathbb{R}$, is uniformly bounded.
4. The family $\{\mathcal{M}(t) : t \in \mathbb{R}\}$ exponentially pullback attracts bounded subsets of X; that is, there exists a positive constant $\omega > 0$ such that for every bounded subset $D \subset X$ and for any $t \in \mathbb{R}$,

$$\lim_{s \to +\infty} e^{\omega s} dist_H(U(t,t-s)D, \mathcal{M}(t)) = 0$$

The paper is divided into two parts. In both cases, the well-posedness of the problem and suitable absorbing families allow to apply two different theorems of existence of pullback exponential attractors.

A first setting involves a translation bounded force $h \in L^2_b(\mathbf{R}; H^{-1}(\Omega))$ and certain nonlinearities. The existence of the pullback exponential attractor for this problem will follow the structure of the main result of [2].

The second block assumes a more general growth for the force. Namely it is allowed to grow exponentially fast in the past. This implies that the absorbing family also satisfies the same type of growth. This better generality allows us to apply the result on the existence of a pullback exponential attractor obtained in [3].

This is a work in collaboration with Professor Pedro Marín Rubio.

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Eigenvalues and Maximum Principle Results for Nonlocal Diffusion Problems

A. A. Casado Sánchez[1], B. M. Molina Becerra[2] and C. A. Suárez *

Let $\Omega \subset \mathbb{R}^N$ be a bounded open set, and let $K \in C^0(\overline{\Omega} \times \overline{\Omega}; \mathbb{R}^{M\times M})$. We study a class of nonlocal problems that include an integral term of the form

$$\int_\Omega K(x,y)u(y)\,dy,$$

where the kernel K models the influence of the values of u at distant points. Such operators naturally arise in population dynamics, where $u(x)$ may represent the population density at location x, and the integral term accounts for the individuals moving from one point to another across the habitat.

Our main objective is to establish the existence of nonnegative continuous solutions to such systems using the method of sub- and supersolutions, following an approach similar to the one in [1], [2], and [3].

To this end, we construct suitable ordered bounds $\underline{u}$ and $\overline{u}$ and prove the existence of a solution u satisfying $\underline{u} \leq u \leq \overline{u}$. A crucial part of the analysis involves the study of eigenvalues.

Furthermore, we prove a strong maximum principle showing that, under suitable assumptions, any nonnegative solution must be strictly positive unless it vanishes identically. These results extend classical tools from elliptic PDE theory to a nonlocal, vector-valued setting.

Finally, we present numerical simulations that illustrate the applicability of our theoretical results.

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Temporal Decay for Non-Newtonian Micropolar Fluids

Felipe W. Cruz[1], María A. Rodríguez-Bellido[2] and Marko A. Rojas-Medar[3]

In this contribution, we study the long time behavior of weak solutions for a class of non-Newtonian micropolar fluids in $\mathbb{R}^2$. Moreover, we investigate the asymptotic behavior of these solutions by comparing them to the solutions of the linear part. More precisely, we are interested in studying the following Cauchy problem:

$$\text{(1)}\qquad \begin{cases} \boldsymbol{u}_t + (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} - (\mu+\chi)\Delta\boldsymbol{u} + \nabla p \\ \quad -2\nu\,\mathrm{div}(|e(\boldsymbol{u})|^{q-2}e(\boldsymbol{u})) = 2\chi\,\mathrm{rot}\,\omega, \\ \mathrm{div}\,\boldsymbol{u} = 0, \\ \omega_t + (\boldsymbol{u}\cdot\nabla)\omega - \gamma\Delta\omega + 4\chi\omega = 2\chi\,\mathrm{rot}\,\boldsymbol{u}, \\ (\boldsymbol{u},\omega)\big|_{t=0} = (\boldsymbol{u}_0,\omega_0), \end{cases}$$

where $q \geq 3$ is a given constant (fixed, but otherwise arbitrary), and $\boldsymbol{u}(\boldsymbol{x},t) \in \mathbb{R}^2$, $p(\boldsymbol{x},t) \in \mathbb{R}$, and $\omega(\boldsymbol{x},t) \in \mathbb{R}$ are the unknowns denoting the linear velocity, hydrostatic pressure, and angular velocity, respectively. The positive constants μ, ν, χ, and γ are related to viscosity. Furthermore, we denote $\boldsymbol{u}_0$ and ω_0 as the initial data for the linear velocity and the angular velocity, respectively, all assumed to be in $L^2(\mathbb{R}^2)$ and such that $\mathrm{div}\,\boldsymbol{u}_0 = 0$ in distributional sense. Next, we will present the concept of a weak solution to the IVP (1).

Definition 16 A weak solution to the system (1) is any pair $(\boldsymbol{u},\omega)$ that satisfies:

$$\boldsymbol{u},\,\omega \in L^\infty\big(0,\infty;\boldsymbol{L}^2(\mathbb{R}^2)\big) \cap L^2\big(0,\infty;\boldsymbol{H}^1(\mathbb{R}^2)\big),$$

$$\boldsymbol{u} \in \cap L^q\big(0,\infty;\boldsymbol{W}^{1,q}(\mathbb{R}^2)\big),$$

with $(\boldsymbol{u},\omega)(0) = (\boldsymbol{u}_0,\omega_0)$ and which satisfies the equations weakly in $\mathbb{R}^2\times(0,\infty)$. Additionally, for $s=0$ and almost every $s>0$, we have:

$$\|(\boldsymbol{u},\omega)(t)\|_{\boldsymbol{L}^2}^2 + \int_s^t \{2\mu\|\nabla\boldsymbol{u}(\tau)\|_{\boldsymbol{L}^2}^2 + 4\nu\|e(\boldsymbol{u})(\tau)\|_{\boldsymbol{L}^q}^q\}d\tau$$
$$+2\gamma\int_s^t \|\nabla\omega(\tau)\|_{\boldsymbol{L}^2}^2 d\tau \leq \|(\boldsymbol{u},\omega)(s)\|_{\boldsymbol{L}^2}^2, \qquad \forall\, t\geq s.$$

Our main results are as follows.

Theorem 17 *Let $\boldsymbol{u}_0 \in \boldsymbol{L}^1(\mathbb{R}^2)\cap\boldsymbol{L}^2_\sigma(\mathbb{R}^2)$, $\omega_0 \in L^1(\mathbb{R}^2)\cap L^2(\mathbb{R}^2)$, and $q\geq 3$. Then for a weak solution $(\boldsymbol{u},\omega)$ of (1), we have that*

$$\|\boldsymbol{u}(t)\|_{\boldsymbol{L}^2} \leq C(t+1)^{-\frac{1}{2}}, \qquad \forall\, t\geq 0,$$
$$\|\omega(t)\|_{\boldsymbol{L}^2} \leq C(t+1)^{-1}, \qquad \forall\, t\geq 0.$$

The next result states that the solutions to equations (1) are asymptotically equivalent to the solutions of the associated linear problem with the same initial data.

Theorem 18 *Let $(\boldsymbol{u},\omega)$ be a weak solution of problem (1), and $(\overline{\boldsymbol{u}},\overline{\omega})$ the solution to the linear part with the same initial data $\boldsymbol{u}_0 \in \boldsymbol{L}^1(\mathbb{R}^2)\cap\boldsymbol{L}^2_\sigma(\mathbb{R}^2)$ and $\omega_0 \in L^1(\mathbb{R}^2)\cap L^2(\mathbb{R}^2)$. Then, for $q\geq 3$, we have:*

$$\|\boldsymbol{u}(t)-\overline{\boldsymbol{u}}(t)\|_{\boldsymbol{L}^2} \leq C(t+1)^{-1}\ln(t+1), \qquad \forall\, t\gg 1,$$
$$\|\omega(t)-\overline{\omega}(t)\|_{\boldsymbol{L}^2} \leq C(t+1)^{-\frac{3}{2}}\ln(t+1), \qquad \forall\, t\gg 1.$$

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Abstract Cauchy problems via generalized ODEs

Lucas Henrique Destro de Toledo[1], Everaldo de Mello Bonotto* and Tomás Caraballo Garrido[2]

Regarding linear evolution equations - extensively studied on [1] and [3] - the autonomous case in Banach space is covered by semigroups:

Definition 19 A family $\{T(t)\}_{t\in\mathbb{R}_+} \subset L(X)$ is called a bounded linear strongly continuous semigroup (or simply C^0-semigroup) if $T(0) = I_X$, $T(t+s) = T(t)T(s)$ for all $t, s \geqslant 0$, and $\lim_{t\to 0^+} T(t)x = x$, for all $x \in X$.

Additionally, the operator $\mathcal{A} : D(\mathcal{A}) \subset X \to X$ given by

$$\mathcal{A}x = \lim_{t\to 0^+} \frac{T(t)x - x}{t}$$

is the (uniquely correspondent) generator of $\{T(t)\}_{t\in\mathbb{R}_+}$.

Considering the abstract Cauchy problem

$$\text{(1)} \qquad \begin{cases} \dot{x}(t) + \mathcal{A}x(t) = f(t), & t > 0, \\ x(0) = x_0 \in X \end{cases}$$

where X is a Banach space, $\mathcal{A}$ the infinitesimal generator of a C^0-semigroup $\{T(t)\}_{t\in\mathbb{R}_+}$ and $f : \mathbb{R} \to X$ is the non-homogeneous therm - also called as a perturbation - it was natural to assume locally absolutely integrable ($f \in \mathcal{L}^1_{loc}(\mathbb{R}_+, X)$) in order to define mild solutions since evolution equations theory lies on Bochner-Lebesgue's integration theory.

Definition 20 Let $-\mathcal{A} : D(\mathcal{A}) \subset X \to X$ be the generator of a C^0-semigroup $\{T(t)\}_{t\in\mathbb{R}_+}$ and $f \in \mathcal{L}^1_{loc}(\mathbb{R}_+, X)$. The function $u \in C^0([0,T], X)$ given by

$$u(t) = T(t)x_0 + \int_0^t T(t-\tau)f(\tau)d\tau,$$

is called a mild solution for 1.

On the other hand, in nature, we can expect locally non-absolutely integrable perturbations due to high oscillations or many discontinuities. In fact, there are integrals - such as Henstock's - that are capable of handling such equations.

Definition 21 Let $f : [a,b] \to X$ be a function. We say that f is Henstock integrable over $[a,b]$ ($f \in H([a,b], X)$, if there exists an associated function $F : [a,b] \to X$ such that for every $\epsilon > 0$, there exists a gauge $\delta : [a,b] \to (0,+\infty)$ such that for every division $d = \{(\tau_i, [s_{i-1}, s_i])\}_{i=1}^{|d|}$ of $[a,b]$, in which $\tau_i \in [s_{i-1}, s_i] \subset (\tau_i - \delta(\tau_i), \tau_i + \delta(\tau_i))$ for each $i \in \{1, \ldots, |d|\}$, we have

$$\sum_{i=1}^{|d|} \| f(\tau_i)(s_i - s_{i-1}) - [F(s_i) - F(s_{i-1})] \| < \epsilon.$$

This concept extends the definition of mild solutions to Henstock mild solutions

Definition 22 Let $-\mathcal{A} : D(\mathcal{A}) \subset X \to X$ be the generator of a C^0-semigroup $\{T(t)\}_{t\in\mathbb{R}_+}$, $w > 0$, and $f : [0,w) \to X$ be a function such that $p_t(\cdot) = T(t-\cdot)f(\cdot) \in H([0,t], X)$ for every $t \in [0,w)$. The function $u \in C^0([0,w), X)$ given by

$$u(t) = T(t)x_0 + \int_0^t T(t-\tau)f(\tau)d\tau,$$

where the above integral is in the sense of Henstock, is called a Henstock mild solution of system (1).

The main objective of this work is dealing with linear evolution equations of the form (1) with non-absolutely integrable perturbations. In order to do this, we approach the problem within a suitable theory of differential equations that deals with Kurzweil and Henstock non-absolutely integrals: the theory of generalized ordinary differential equations, or simply GODEs - see [2], [4].

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Reduced Order Model for Time Splitting Schemes

Mejdi Azaïez[1] , Tomás Chacón[2] , Carlos Núñez[†], Samuele Rubino[†]

This poster presents recent advances in reduced order modeling(ROM) for incompressible fluid dynamics problems, focusingon time-splitting schemes and Proper Orthogonal Decomposition(POD). Two contributions are highlighted. First, we proposea POD-Galerkin ROM for the Navier Stokes equationswith open boundary conditions, where a low-dimensional PDEis solved on the boundary to impose an equivalent Dirichletcondition on the pressure. We compare a standard projectionbasedROM with a hybrid model that integrates data-drivenRadial Basis Functions (RBF), demonstrating improved flexibilityand accuracy in complex geometries such as bifurcatedtubes and flow past a cylinder. Second, we explore a first-orderROM formulation for the unsteady Stokes equations based onthe pressure-correction Goda scheme. Here, different innerproducts are used to construct reduced bases, enabling explicitreduced formulations for both velocity and pressure. Stabilityand error analyses support the robustness of the approach, and numerical tests confirm its effectiveness, including for parametrized Navier Stokes problems. Together, these resultsillustrate the potential of POD-ROMs for efficient and accuratesimulation of complex, time-dependent incompressible flows.

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Mathematical modeling of Neuroblast Migration towards the Olfactory Bulb

Daniel Acosta-Soba[1], Carmen Castro[2], Noelia Geribaldi-Doldán[3], Francisco Guillén-González[4], Pedro Nunez-Abades[5], Noelia Ortega-Román*, Patricia Pérez-García[6], and J. Rafael Rodríguez-Galván*

This work is focused on mathematical modeling of the migration of neuroblasts (immature neuron cells) along the Rostral Migratory Stream in rodent brains [1]. According to our model it is determined mainly by attraction forces to the olfactory bulb, and also by the heterogeneous mobility of neuroblasts in different regions of the brain. Carefully identifying them as solutions to partial differential equations allows us to determine the movement of neuroblasts in a realistic fashion. We develop numerical schemes where the application of novel discontinuous Galerkin methods [3] allows to maintain the properties of the continuous model. We present some successful computer tests including parameter adjustment to fit real data.

The model of neuroblast evolution

Let $\Omega \subset \mathbb{R}^2$ be an open set representing a rodent brain, with boundary $\partial\Omega$ and $T > 0$. We consider the following problem modelling the evolution of the density of neuroblasts:

$$\begin{aligned} u_t + \chi \nabla \cdot (u \nabla \mathcal{O}) + \alpha\, u - \gamma\, u\, \mathbb{1}_{NZ} &= 0 \quad \text{in } \Omega \times (0,T), \\ u &= 0 \quad \text{on } \partial\Omega \times (0,T), \\ u(0) &= u^0 \quad \text{in } \Omega, \end{aligned}$$

where $\mathcal{O} = \mathcal{O}(x,y) \in \mathbb{R}$ is a potential function whose gradient $\nabla\mathcal{O}$ models the attraction exerted by the olfactory bulb.

The olfactory bulb chemoattractant function

We consider the next source term, defined as a Gaussian function centered at the middle point of the olfactory bulb:

$$f_{\mathcal{O}}(x,y) = e^{-((x-x_O)^2+(y-y_O)^2)/\sigma^2}.$$

And we consider the following problem: find $\mathcal{O} \in H^1(\Omega)$ as the solution to the boundary-value problem

$$\begin{cases} \mathcal{O} - \nabla \cdot (\mu_O \nabla \mathcal{O}) = f_{\mathcal{O}} & \text{in } \Omega, \\ \mathcal{O} = f_{\mathcal{O}} & \text{on } \partial\Omega. \end{cases}$$

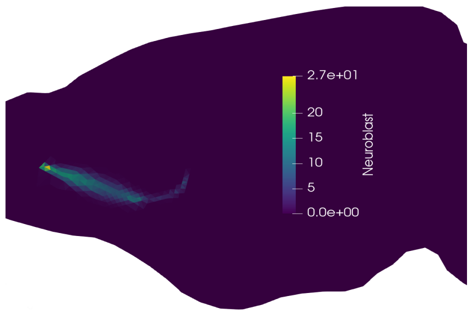
where μ_O is a piecewise constant function which will be used as an anisotropic diffusion coefficient.

Numerical tests and adjustment to real data

After an optimization process, in which we use real data obtained from rodent brain images, we determine an adequate initial condition in realistic brain domains. This initial condition comes from a similar stationary problem. Also, we obtain the following parameters for the neuroblast evolution model:

$$\alpha = 1.951\cdot 10^{-1}, \qquad \chi = 2.241\cdot 10^{-2} \quad \text{and} \quad \gamma = 3.548.$$

The relative quadratic error made by our simulation of the entire migration process with respect to the real data is: $E = 0.28$. We have represented the neuroblasts migration process for the optimal parameters:



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The stochastic TR-BDF2 and ICC model

T. Caraballo[1], M. Gómez-Mármol* and I. Roldán*

Ordinary differential equations and their numerical analysis aim to model numerous real-world phenomena. While numerical analysis is well-established in the deterministic context, the introduction of stochastic terms allows us to capture the inherent randomness present in natural processes, unlocking characteristics unattainable in deterministic problems.

The numerical resolution of the stochastic systems required tailored development of methods, along with a comprehensive understanding of error analysis, convergence rates, and other related concepts [2]. This approach is crucial for comprehending the stochasticity and can be supported by the extensive literature devoted to numerical analysis in the deterministic case.

In this talk, our focus is the development and numerical examination of the stochastic version of the TR-BDF2 method:

$$\begin{cases} \text{Given } y_n, \forall n = 0, \ldots, N-1, \\ y_{n+\gamma} = y_n + \dfrac{1}{2}\left(a\left(t_n, y_n\right) + a\left(t_{n+\gamma}, y_{n+\gamma}\right)\right) h\gamma, \\ y_{n+1} = \gamma_3 y_{n+\gamma} + \left(1-\gamma_3\right) y_n + a\left(t_{n+1}, y_{n+1}\right) h\gamma_2. \end{cases}$$

We delve into its performance characteristics, focusing on preserving the second-order accuracy of the deterministic counterpart and conduct stability analysis in the presence of stochastic terms. Additionally, we present validation tests in academic stiff scenarios to assess the practical applicability of the theoretical results obtained.

In particular, we are interested in studying a stochastic formulation of the following fast-slow ICC model [1]:

$$\begin{cases} \dot{x} = \tau\left(-y + \alpha x - x^3 - \dfrac{\mu z}{z + z_0} + I_{ext}\right), \\ \dot{y} = \tau\varepsilon\left(x + a_1 y + a_2\right), \\ \dot{z} = \tau\varepsilon\left(\dfrac{\lambda}{1 + \exp\left(-\rho\left(x - x_{on}\right)\right)} - \dfrac{z - z_b}{\tau_z}\right). \end{cases}$$

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Optimal Control for Degenerate Chemotaxis Models

Sarah Serhal[1 2], Georges Chamoun [3], Mazen Saad* and Toni Sayah†

In this presentation, we study a controlled chemotaxis model arising from biology, formulated as a parabolic-parabolic system with degenerate diffusion and a control term acting on the concentration of the chemoattractant. The objective is to determine an optimal control on the chemoattractant concentration in order to reach a target cell distribution, particularly in the context of cancer treatment, where the goal is to minimize cancer cells.

This problem is formulated as a constrained optimization problem, where the constraints are defined by the weak solutions of the controlled model. Unlike classical approaches based on strong regularity assumptions for the state solutions, our formulation focuses on weak solutions. This approach allows us to guarantee the existence of an optimal control while simplifying both the mathematical analysis and the formulation of the adjoint system, especially in the case of degenerate diffusion. Previous studies (see [1, 2]) have not addressed the specific issue of diffusion degeneracy, which represents a major difficulty in our framework. Our main objective is therefore to study the existence of weak solutions for the optimal control problem associated with chemotaxis models involving a diffusion function with two-sided degeneracy.

We first prove the existence of solutions for the controlled model using a semi-discretization in time inspired by [3]. We show that the semi-discrete solution satisfies the maximum principle for any control, where the positive part is treated explicitly and the negative part implicitly.

We then establish the existence of an optimal control and its corresponding adjoint system using the Lagrange multiplier method. Unlike classical cases, this multiplier satisfies a backward parabolic equation, which is well-posed only when the final time is fixed. The equation associated with the Lagrange multiplier itself presents a degenerate structure, posing additional analytical challenges. We therefore define a regularized system to overcome the degeneracy under certain regularity conditions (see [4])

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